MINIMAL MORSE FUNCTIONS VIA THE HEAT EQUATION IN LOCALLY HOMOGENEOUS RIEMANNIAN MANIFOLDS

by

JHON WILLINGTON BERNAL VERA

Directed by

CARLOS ALBERTO CADAVID MORENO

EAFIT University
Department of Mathematical Sciences

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Abstract

Let (M, g) be a riemannian manifold is connected and closed (compact without boundary), and let Δ_g be the Laplace-Beltrami operator determined by g. The heat equation on (M, g) is the differential equation $\frac{\partial u}{\partial t} = \Delta_g u$ where $u : M \times [0, +\infty) \to \mathbb{R}$. For each function $f : M \to \mathbb{R}$ in $L^2(M, g)$ there exists a unique solution u of the heat equation such that $u(\cdot, 0) = f$. It has been conjectured that

If (M, g) is locally homogeneous, i.e. each pair of points p, q in M, have isometric neighborhoods, then there exists an open dense subset S of $L^2(M, g)$, with the property that for each $f \in S$ there exists a real $T_f > 0$ such that if $t \geq T_f$, the function $u(\cdot, t) : M \to \mathbb{R}$ is Morse and has a number of critical points less than or equal to the number of critical points of any other Morse function on M.

It is natural to start the study of this conjecture examining a collection as rich as possible of examples of locally homogeneous riemannian manifolds of different dimensions. Examples in dimension 3 are particularly adequate for testing the conjecture, because they are well known, and also because they are more varied and less trivial than manifolds in dimensions 1 and 2.