# Does Liquidity Risk Premium Affect Optimal Portfolio Holdings of U.S Treasury Securities?

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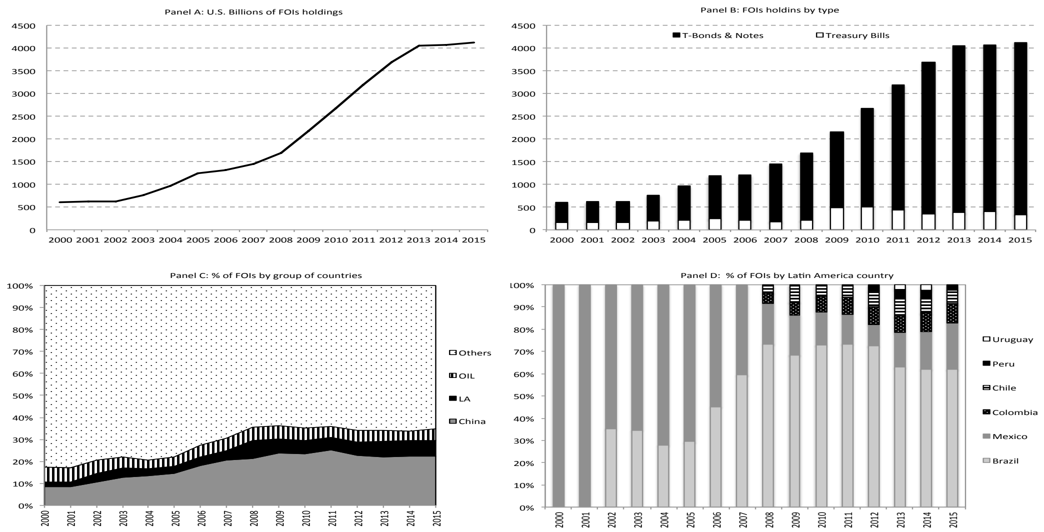
**Abstract**

Over the last fifteen years, foreign official holdings of U.S. Treasuries reached unprecedented levels, being central banks, especially in developing countries, the first to significantly increase their holdings; which calls for an analysis of risk-factor portfolio allocation to U.S. government bonds. This paper focus on examining how changes in the liquidity differential between nominal and TIPS yields influences optimal portfolio allocations in U.S. Treasury securities. Based on a non-parametric estimation technique and comparing the optimal allocation decisions of mean-variance and CRRA investor, when investment opportunities are time varying, I present evidence that liquidity risk premium is a significant risk-factor in a portfolio allocation context. In fact, I find that a conditional allocation strategy translates into improved in-sample and out-of sample asset allocation and performance.
**Keywords:** Liquidity risk, optimal portfolio allocation, bond risk premia, non-parametric estimation, foreign exchange reserves.
**JEL classification:** C13, C52, G11, G32,

## 1 Introduction

Foreign exchange reserves, as a key part of the asset side of the central bank's sheet, play a fundamental role in monetary management. Almost all foreign exchange reserves are held in five currencies: the U.S. dollar, the euro, the Japanese yen, the British pound, and the Swiss franc; accounting dollar reserve holdings for 61 percent of the total at the end 2014 (BIS). The dollar commands a high share in global reserves because the deep and highly liquid market for the U.S. Treasury bonds. Indeed, over the last fifteen years, foreign official holdings of U.S. Treasuries reached unprecedented levels, increasing from \$600 billion in January 2000 to about \$4.142 billion in January 2015. Foreign Official Institutions (FOIs), such as central banks, were the first to significantly increase their holdings, as part of their reserve accumulation policies. The pattern of the foreign demand for U.S. Treasury securities is showed in Figure 1, panel A. Panel B shows that on average 82% of the FOIs holdings correspond to Treasury notes (with maturities between 1 and 10 years) and T-bonds (more than 10-year maturity), while 18% is held in bonds with less than 1-year maturity.

Figure 1: Foreing holdings of U.S. Treasury securities




The data in each panel include foreign holdings of U.S. Treasury bills, bonds, and notes reported every January under the Treasury International Capital (TIC) reporting system for the years 2000 to 2015. OIL represents Oil Exporters, which include Ecuador, Venezuela, Indonesia, Bahrain, Iran, Iraq, Kuwait, Oman, Qatar, Saudi Arabia, the United Arab Emirates, Algeria, Gabon, Libya, and Nigeria. L.A. represents Latin American countries (except Ecuador and Venezuela).

Most of this growth is accounted for by the emerging market economies, where besides China, Latin American countries have played an important role. As Panel C shows, over the past fifteen years, China has steadily increased its U.S. Treasury holdings to become the largest U.S. holder, with about 30% of the total. Similarly, Latin American countries noticeably increased the level of international reserves. Indeed, official holdings of U.S. Treasuries increased from $17 billion in January 2000 to almost $413,9 billion in January 2015, representing 10% of the total. Holdings by the Oil Exporters (which include Venezuela and Ecuador) have also grown in the past few years, representing 7% of the total in January 2015. Finally, Panel D depicted the market share of the Latin American major foreign holders of U.S. Treasury securities by year. Until 2007 only Mexico and Brazil appeared in the group of major foreign holders, however since 2008 other countries, such as Colombia, Chile, Peru and Uruguay, have significantly increased their holdings. In such a context, the foreign officials' portfolio of U.S. Treasury securities became in a "self-insurance" strategy against potential external shocks, or resulted as a by-product of interventions by Latin American central banks in foreign-exchange markets, especially in the context of surges of capital inflows that characterized part of this period.

Investment portfolios traditionally have been constructed with a focus on what asset classes to invest in and how much to invest in each. Recent research, however, has shown that focusing on risk-factor allocations, rather than asset class allocations, can result in better risk-adjusted portfolio performance. This paper contributes to this debate. In particular, I focus on examining how changes in the liquidity differential between nominal and TIPS yields influences optimal portfolio allocations in U.S. Treasury securities. There are a series of ways in which this study contributes to the literature. First, it incorporates financial information (liquidity premium) in an asset allocation context, and shows how this can be of significance for both a mean-variance and a CRRA investor. Second, it focuses on a bond portfolio choice that is relatively unexplored in the literature, since the majority of the studies on asset allocation examine stock-only portfolios. Additionally, portfolio of U.S. Treasury securities are singularly important for central banks, especially in developing countries. Third, I examine portfolio choice among multiple government bonds with different maturities. More so, I consider both the U.S. Treasury bonds and Treasury Inflation Protected Securities (TIPS) in the investor's asset menu.

Government debt of the United States includes Treasury bonds and TIPS. The U.S. Treasury Department typically has issued debt in the form of Treasury bonds, including T-bills, T-notes and T-bonds (according with its maturity). These securities, simply called Treasuries, are widely regarded to be the safest investments because the deep and highly liquid market added to its lack significant default risk. Therefore, it is no surprise that investors turn to U.S. Treasuries during times of increased uncertainty as a safe haven for their investments (flight to quality). In addition to Treasuries, in 1997, the U.S. Treasury Department started its Treasury Inflation-Protected Securities (TIPS) program. The program is intended to provide investors with protection against inflation.[1] TIPS has shown a consistent growth since its inception in 1997. In fact, the market capitalization has grown by more than thirty times, from $33 billion dollars in 1997 to over $1.200 billion in 2013. However, it has been characterized by be less liquid than nominal Treasury bond market. As a consequence, the lack of liquidity is thought to result in TIPS yields having a liquidity premium relative to nominal securities.

Liquidity risk in Treasury markets, arise from the fact that investor may need to make portfolio adjustments due to some unforeseen events, after the initial auction or before the maturity of a Treasury security. Thus investors care about the likely costs associated with such trading, and in response, demand a higher yield to compensate the cost to buy or sell the security in a secondary market. Even though, nominal Treasuries and TIPS have similar trading costs (such as brokerage fees and commissions), the cost related to the ease and convenience of trading is not the same for both securities, and it might be related to the differences in liquidity market conditions. Hence, the additional yield to compensate the incremental risks and higher costs of trading is referred as liquidity risk premium. The existence of this liquidity premium in TIPS yields has been well documented in the academic literature by Campbell et al. (2009), Dudley et al. (2009), Christensen and Gillan (2011), Gurkaynak et al. (2010), Pflueger and Viceira (2012), Gomez (2015), among others.

Focusing on a conditional allocation technique and comparing the optimal allocation decisions of mean-variance and CRRA investor, when investment opportunities are time varying, I present evidence that liquidity risk premium is an significant risk-factor in a portfolio allocation context. Throughout this paper, I assume that the investor makes decisions in real terms where the investment horizon is one-month, one-quarter and one-year. I only consider a short-term investor in the empirical analysis. The reason for this is related to the fact that for a buy-and-hold long-term investor, whose investment horizon perfectly matches the maturity of the bond, TIPS offer full protection against inflation if held until maturity.[2] Similarly, an investor who adopts a buy-and-hold strategy for TIPS mitigates risk

[1] TIPS help to guard against inflation by adjusting the face value with changes in the rate of inflation. Interest is then paid on the adjusted face value of the bond.

[2] TIPS are a useful hedge against inflation, but they do not guarantee a real rate of return. This is because the mechanics of adjusting for inflation for TIPS limit the exactness of the inflation adjustment and allow only approximate inflation hedges especially at high inflation levels. In fact, for TIPS, the reference price index is the non-seasonally adjusted CPI-U, and the indexation lag is three months. Therefore, TIPS operate with an indexation lag of three months. In other words, it takes three months from the incidence of price inflation (the month when a reference index reading is

arising from illiquidity, given that he/she does not face higher costs of buying or selling the bond before it reaches maturity. However, TIPS are currently issued with only a few specific maturities: 5-year, 10-year and 30-year, therefore the investment horizon over which I consider investors who hold assets does not match the maturity of any outstanding TIPS.[3] Hence, I study a short-term investor who maximizes real wealth but is not able to invest in a risk-less asset in real terms (given that TIPS are a risky asset both in nominal and in real terms), and also faces liquidity risk. Notice, however, that a short-term investor benefits from the availability of TIPS in terms of a wider investment opportunity set that allows an increase in the returns per unit of risk, investing even a small fraction of his wealth in TIPS (Cartea et al. (2012)).

The investor's problem is to choose optimal allocations to the risky asset as a function of predictor variable: the TIPS liquidity premium. As risky assets, I consider equally weighted bond portfolios on short-term bonds (1 to 10 years maturity); and on long-term bonds (11 to 20 years maturity), each of them are computed for Treasury bonds and for TIPS. I identify the liquidity component in TIPS yields through the difference between inflation-linked and nominal bond asset swap spreads. This difference is capturing of the relative financing cost, the specialness and the balance sheet cost of TIPS over nominal Treasury bonds. These characteristics influence the ease of liquidating some securities and the attractiveness by which to hold them with respect to others. Therefore, this is a market-based measure of the market perception (current and expected) of relative liquidity in the bond market. The particular choice of this measure for the liquidity premium is motivated by the fact that: *i*) it is highly correlated with other measures of the TIPS liquidity premium available in the literature, which suggests that they are all capturing similar information about the liquidity differential between nominal and TIPS yields; *ii*) U.S. bond excess returns can be predicted by this liquidity measure (see Gomez (2015)); and *iii*) it is a market-based measure of liquidity which is straightforward to compute.

In summary, I consider the portfolio policy of an investor who is able to invest in only one risky asset, and I differentiate various portfolio allocation problems: first, where the investor chooses between the portfolio of short-term or long-term Treasury bonds and a risk-free asset; and second, where the investor chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. I also study an investor with mean-variance (MV) and constant relative risk aversion (CRRA), with different degrees of risk aversion, in order to test the sensitivity of the optimal portfolio choice to the higher moments.

I make use of an econometric framework based on a portfolio choice problem of a single period investor, where the investor's problem is set up as a statistical decision problem, with asset allocations as parameters and the expected utility as the objective. The allocations are estimated by direct maximization of expected utility proposed by Brandt (1999). A number of key results emerge from this analysis. First, the liquidity premium seems to be a significant determinant of the portfolio allocation of U.S. government bonds. In fact, conditional allocations in risky assets decrease as liquidity conditions worsen. In particular, an increase in the liquidity differential between nominal and TIPS bonds leads

recorded) until it is incorporated into the coupon payment of the inflation-linked bond. Consequently, the indexation lag affects how well TIPS compensate for contemporaneous inflation, and prevents TIPS from guaranteeing a specified real return.

[3] TIPS bonds have been offered in 5-, 10-, 20-, and 30-year denominations. However, TIPS that have less than one year remaining to maturity are not easy to find in the secondary market, given that they have extremely high transaction costs.

to lower optimal portfolio allocations for nominal Treasury bonds, and also to lower optimal portfolio allocations in TIPS, but at different levels of liquidity. Additionally, the effect of liquidity is a decreasing function of investment horizons, in the sense that for the same degree of risk aversion the investor reacts less abruptly to an increase in the liquidity premium when he/she has a longer investment horizon. Furthermore, as the investment horizon becomes longer, the smaller the optimal portfolio weight, and so, the less is invested in the risky asset.

The above conclusions are not determined by the level of risk aversion or the investors preferences. The relation between optimal portfolio weights and the liquidity premium remains the same for different values of risk aversion, and also across investor preferences. These characteristics mainly change the level of the portfolio function, having a small impact on the shape of the function. In addition, results do not depend on a particular choice of the maturity of the liquidity premium (similar results are found when considering 10-year or 20-year liquidity premium), nor on a specific way to proxy liquidity (I have similar results with both liquidity premium measures).

From the standpoint of practical advice to U.S. Treasury security investors, a final natural question to ask is whether or not a conditional strategy translates into improved (in and out-of-sample) asset allocation and performance. To answer this question, I compare the performance of the optimal portfolio choices of two investors: one investor who makes portfolio allocations conditional upon observing a particular liquidity signal (conditional strategy); and the other who ignores any change in liquidity in making his/her portfolios allocation choices (unconditional strategy). I conclude that the conditional strategy outperforms the unconditional strategy, improving not only the in-sample, but the also out-of-sample asset allocation and performance.

The rest of the paper is organized as follows. Section 2 defines the conditional portfolio choice problem and presents the non-parametric estimation technique used. I describe the data and provide some basic statistics in Section 3. Section 4 presents the empirical results for different bond portfolios, different types of investors and different investment horizons. Section 5 concludes.

## 2 The conditional optimal portfolio problem

The traditional problem of optimal portfolio choice considers an investor which maximizes the conditional expected utility of next period's wealth under a budget constraint. Merton (1969) provides the solution, where the investor can trade continuously in a finite set of stocks and bank account. However, given that the stocks and bonds differ in many ways, the theory of portfolio management does not apply as it stands to bond portfolios (see Ekeland and Taflin (2005) for a discussion of this point). For the bond market, Schroder and Skiadas (1999), Ekeland and Taflin (2005), Ringer and Tehranchi (2006) and Liu (2007) have studied this problem using a theoretical approach. In particular, Ekeland and Taflin (2005) and Ringer and Tehranchi (2006) set up, and solve the problem of managing a bond portfolio by optimizing (over all self-financing trading strategies for a given initial capital), the expected utility of the final wealth. Thus, optimal portfolio at time $t$ is a linear combination of self-financing instruments, each one with a fixed time to maturity. Under this set up the value of the portfolio changes only because the bond prices change. Price bonds behave like price stocks, that is, it depends only on the risk it carries and not on time to maturity.

The impact of inflation on portfolio choice also has also been considered in the literature. An initial extension of the Markowitz problem was introduced in the 1970s by Biger (1975), Friend et al. (1976), Lintner (1975) and Solnik (1978), among others. Intertemporal portfolio choice problem under inflation risk was studied by Campbell and Viceira (2001) in discrete time, and by and Brennan and Xia (2002) in continuous time. Both works tell us that a long-term, risk-averse investor prefers the indexed bond or a perfect substitution of indexed bond in order to hedge against the inflation risk. However, in these papers all relevant state variables are assumed observable and the probability distributions of all processes are assumed known. Bensoussan et al. (2009) and Chou et al. (2010) relax that restriction by assuming that the expected inflation rate is unobservable to the investor.

Most of the existing studies on portfolio choice (with or without inflation risk), focus on stock-only portfolios (Viceira and Campbell (1999), Barberis (2000), Wachter (2002)), or examine the stock-bond mix portfolio choice (Munk et al. (2004)). Given the extensive literature for equity markets, it is surprising to note that no effort has been undertaken to examine the influence of liquidity in government bond portfolio choice. Filling this gap is one contribution of this paper. To follow, I define the investor's maximization problem, describe the conditioning information, and finally, introduce the estimation technique.

## 2.1 Investor utility maximization

### 2.1.1 Portfolio choice without inflation

Ekeland and Taflin (2005) and Ringer and Tehranchi (2006) express the solution of optimal portfolio choice as portfolios of self-financing trading strategies which naturally include stocks and bonds. In particular, they fix a utility function $u$ and a planning horizon $T > 0$, and consider the functional $J(\varphi) = \mathbb{E}^{\mathbb{P}}[u(W_T^{\varphi})]$ where $W_T^{\varphi}$ is the accumulated wealth at time $T$ generated by the self-financing trading strategy $\varphi$. The goal is to characterize the strategy that maximizes $J$.

Following on from this literature, I consider the problem of optimal portfolio choice when the traded instruments are a set of risky bonds and a risk-less bond. In particular, and without loss of generality, I consider a bond market where only zero-coupon bonds are available. Fixing a utility function $u(W_{t+1})$ and a planning horizon $T > 0$, I consider an investor who maximizes the conditional expected utility of next period's wealth, subject to the budget constraint:

$$\begin{aligned} &\max_{\alpha_t \in \mathcal{A}(\varphi)} \quad \mathbb{E}[u(W_{t+1}) \mid Z_t] \\ &\text{subject to:} \quad W_{t+1} = W_t[R_{f,t+1} + \alpha_t(R_{b,t+1} - R_{f,t+1})] \end{aligned} \tag{1}$$

where $W_{t+1}$ is the accumulated wealth at time $t+1$ generated by the self-financing trading strategy $\varphi$ (which belongs to the set of admissible self-financing strategies denoted by $\mathcal{A}$), $\alpha_t$ represents the proportion of wealth invested in a risky bond with return $R_{b,t+1}$ and the remaining proportion $1 - \alpha_t$ is invested in risk-free bond with return $R_{f,t+1}$. The expectation is conditional on a state variable $Z_t$. The investor can have three different horizons: one-month, one-quarter or one-year (this represents the difference between $t$ and $t + 1$).

The weight that maximizes the expected utility function is the solution to the following Euler optimality condition

$$\mathbb{E}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha} \mid Z_t\right] = 0. \tag{2}$$

In particular, the solution of the investor's problem is the mapping from the state variable $Z_t$ to the portfolio weights

$$\alpha_t = \alpha(Z_t), \tag{3}$$

and it denotes the portfolio choice of observing a signal $Z_t = z$.

The relation between the portfolio policy and the predictability of individual moments of the returns given the predictor $Z_t$ depends on the specification of the utility function. I consider two types of investor preferences: mean-variance (MV) and power-utility (CRRA) preferences. An investor with mean-variance preferences maximizes

$$\max_{\alpha_t \in \mathcal{A}(\varphi)} \quad \mathbb{E}[W_{t+1} \mid Z_t] - \frac{\gamma}{2}\mathbb{V}[W_{t+1}^2 \mid Z_t], \tag{4}$$

where $\gamma > 0$ represents the coefficient of absolute risk aversion. The investor portfolio policy when the choice includes a risk-free rate is proportional to the conditional mean-variance ratio of the tangency portfolio

$$\alpha_t^{tg} = \frac{1}{\gamma W_t}\frac{\mathbb{E}[R_{t+1}^{tg} \mid Z_t]}{\mathbb{V}[R_{t+1}^{tg}]},$$

where $R_{t+1}^{tg}$ is the return of the tangency portfolio. The reason I consider MV preferences is because it can be stated as a primitive, or can be derived as a special case of expected utility theory. Also, under MV preferences, portfolio weights depend exclusively and analytically on the two first moments of returns, which serve as benchmark case in this study.[4]

I also consider the most popular objective function in the portfolio choice literature, which is an investor with CRRA or power utility. In this case, the investor solves the following problem

$$\max_{\alpha_t \in \mathcal{A}(\omega)} \begin{cases} \mathbb{E}\left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma}\right] & \text{if } \gamma > 1 \\ \mathbb{E}\left[\log(W_{t+1})\right] & \text{if } \gamma = 1 \end{cases} \tag{5}$$

subject to the budget constraint in (1), and where $\gamma > 0$ now measures the coefficient of relative risk aversion. As is well known, unlike mean-variance preferences, CRRA does not permit a closed form solution to the investor's portfolio problem. However, I consider CRRA preferences to be able to test whether or not an investor cares about higher order moments of the return distribution.

[4] Although the limitations of mean-variance analysis are well established in portfolio theory, its relative simplicity and easy intuition contributes to its continued use among investment professionals, in theoretical and empirical studies.

### 2.1.2 Portfolio choice with inflation

In this section, I follow Cartea et al. (2012) who solve the optimal portfolio choice problems for investors concerned with maximizing real wealth. Here, I assume that investors make allocation decisions in real terms, and are worried about the purchasing power of their terminal wealth, and do not suffer from money illusion. As before, I consider the optimal investment allocation of investors who are not worried about what may happen beyond the immediate next period but rather, care about the purchasing power of their wealth.

To avoid exposure to inflation risk, investors can: ($i$) invest in a risk-less asset in real terms; and/or ($ii$) invest in assets that covary with inflation. However, in this empirical analysis I only consider investors who have a maximum investment horizon of 1-year; they cannot find TIPS with this maturity and thus they are not able to invest in a risk-less real asset. Additionally, given that real interest rate changes affect TIPS returns, investors consider TIPS as a risky asset in both nominal and real terms.

An investor with MV or CRRA preferences maximizes the same problem in (4) and (5), respectively, but are now subject to the budget constraint

$$W_{t+1}^R = W_t^R[R_{f,t+1} + \alpha_t(R_{b,t+1} - R_{f,t+1})],$$

where $W_{t+1}^R$ is now the terminal real wealth, and $R_{b,t+1}$ and $R_{f,t+1}$ are real risky and risk-free bond returns, respectively, as already seen.[5] In the absence of a real risk-free asset investors face inflation risk and deal with this through the covariances between the returns of risky assets and inflation. Securities which are correlated with inflation help to hedge against inflation, reducing the portfolio variance in real terms.

## 2.2 Non-parametric estimation

I use the methodology proposed by Brandt (1999) and Ait-Sahalia and Brandt (2001). They apply a standard generalized method of moments (GMM) technique to the conditional Euler equation that characterizes the investor's portfolio choice problem. In particular, it consists of replacing the conditional expectation with sample analogues, computed only with returns realized in a given state of nature where the forecasting variable level is $Z_t = \bar{z}$ (which is liquidity premium). Brandt (1999) suggests estimating the conditional expectation with a standard non-parametric regression. Ait-Sahalia and Brandt (2001) suggest a semiparametric approach to address the issue of which predictors are important for the portfolio choice when a large number of them are available.

Let a neighborhood of $Z$ be $Z \pm h$ for some bandwidth $h > 0$. When the investor is characterized by the power utility, a simple non-parametric estimator of the conditional Euler equation is given by the Nadaraya-Watson estimator, where the moment condition is given by:

$$\hat{\mathbb{E}}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha} \mid Z_t = \bar{z}\right] = \frac{1}{Th}\frac{\Sigma_{t=1}^T\left(\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right)k(Z_t, \bar{z}, h)}{\Sigma_{t+1}^T k(Z_t, \bar{z}, h)} = 0, \tag{6}$$

[5] In this case, the real risk-free bond returns is calculated as $R_{f,t+1} - \pi_{t+1}$, where $\pi_{t+1}$ is the log inflation rate.

where $k(Z_t, \bar{z}, h)$ is the kernel function which is assumed to be Gaussian. I apply exactly identified GMM to equation (2) to obtain $\hat{\alpha}(Z)$ which is a consistent estimate for the unknown optimal portfolio choice $\alpha(Z)$ (See Ait-Sahalia and Brandt (2001) for asymptotic properties of this estimators). The conventional solution to optimize the classical trade-off between variance and bias is to choose a bandwidth of the form: $h = \lambda \sigma_z T^{-1/K+4}$, where $\lambda$ is a constant, $K$ is the number of predictor variables and $\sigma_z$ is the standard deviation of the predictor $Z$ (see Hardle and Marron (1985)).

Finally, the optimal unconditional portfolio weight is compute by applying a standard GMM procedure to the unconditional Euler equation. In this case the moment condition is:

$$\hat{\mathbb{E}}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right] = \frac{1}{T}\Sigma_{t=1}^{T}\left(\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right) = 0, \tag{7}$$

which yields the same results that directly compute weights from equation (3).

# 3 The Data and basic statistics

I am interested in the analysis of the empirical time-series relationship between optimal bond portfolio allocations and alternative measures of liquidity. To that end, I calculate monthly, quarterly and annual holding period returns from daily observations of zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak et al. (2007) and Gurkaynak et al. (2010) for observed bond yields, respectively, available through the Federal Reserve web site. This data set contains constant maturity yields for maturities of 2 to 20 years. I construct equally weighted bond portfolios on short-term bonds (1 to 10 years maturity) and on long-term bonds (11 to 20 years maturity), each of them computed for Treasury bonds and for TIPS, ending up with four risky assets. The sample period is from January 2, 2004 to December 31, 2012.

For the same period, I also collect information on one-year Treasury bills from the Federal Reserve Board statistical releases. Following Ait-Sahalia and Brandt (2001) and Ghysels and Pereira (2008) I assume Treasury bill is risky-free, and I fix the risk-free rate at its historical average. They argue that the constant risk-free rate assumption guarantees that any difference in the optimal portfolio functions across frequencies is solely due to the relation between returns and liquidity. In summary, the asset universe consists of the short-term Treasury bonds (weight $\alpha_{NS}$), the long-term Treasury bonds (weight $\alpha_{NL}$), the short-term TIPS (weight $\alpha_{RS}$), the long-term Treasury bonds (weight $\alpha_{RL}$) and the risk-free assets (weight $\alpha_{rf}$).

I calculate liquidity premium for 10 and 20-years to maturity as the residual spread between TIPS and nominal z-spread asset swaps using daily data from January 2004 to December 2011 I using daily nominal and TIPS $z$-spread asset swaps data from Barclays Live. I denote this measure by $L_{n,t}^{z\text{-}asw}$ and it is going to be the predictor variable $Z_t$. I find that this variable is highly correlated and shares the same dynamic pattern with other measures of relative bond liquidity premium proposed in literature by Christensen and Gillan (2011) and Pflueger and Viceira (2012). Additionally, it is strictly positive for all maturities and shows a peak in late 2008 during the financial crisis.

Table 1 shows descriptive statistics of the liquidity predictor and holding period government bond portfolio returns, for the three investment horizons: one-month, one-quarter and one-year. The first lines in each panel show the mean, standard deviation, skewness and kurtosis for each liquidity measure and returns. By construction, and to facilitate the interpretation of the results, liquidity measure has a mean zero and standard-deviation equal to one (i.e. they have been standardized). Also, there is evidence of fat tails in returns, especially at the shorter investment horizon. This tail risk suggests that the distribution is not normal, but skewed, and has fatter tails. The fatter tails increase the probability that an investment will move beyond three standard deviations. Nominal returns are negatively correlated with liquidity while TIPS returns are positively correlated. This means that as liquidity conditions worsen (higher liquidity premium), TIPS returns rise in order to compensate for the higher risk in bad times.

The following lines show the autocorrelation coefficients for different lags, which do not suggest persistence in most of the variables, especially at any frequency. The last line shows the p-value for the Dickey and Fuller test. The $p$-value for the Dickey and Fuller tests suggest the rejection of the null of a unit root for both short-term and long-term returns, and Christensen and Gillan (2011) 10-years liquidity.

# 4 Empirical results

## 4.1 Unconditional portfolio weights

The goal in this section is to characterize the unconditional portfolio choice which serves as a benchmark for the conditional problem. Table 2 presents estimates of unconditional portfolio choices of investors with MV and CRRA preferences with different risk aversion degrees of $\gamma = 2$, 5, 10 and 20, and for three investment horizons. The entries in each column correspond to a portfolio choice between Treasury bills (assumed as risk-free) and one of the four different equally-weighted portfolio bonds: short-term nominal bonds (NS), long-term nominal bonds (NL), short-term TIPS (RS) or long-term TIPS (RL). That they do not impose short-sell constraints suggests a less realistic environment, mainly because the Markowitz portfolio tends to have very large quantities of individual assets (sometimes unreasonably so), I do not impose this restriction to make my results comparable with previous papers.

Several well-known features of optimal portfolio choice emerge. Consider the mean-variance portfolio choice weights. First, risk aversion affects how much wealth the investor allocates to risky securities instead of to the risk-free Treasury bill. The more risk-averse the investor, the less they will invest in the risky bond, so that long positions in risky bonds goes down with a higher degree of risk aversion. Second, given that this investor is forming his portfolio using only bonds and the risk-free Treasury bill, he/she will not want to short-sell the risky asset but rather will want to buy it on the margin (i.e. $\alpha > 1$). That means investors borrow money at risk-free rates and go long in risky bonds. For instance, an investor with an annual investment horizon and $\gamma = 20$ borrows 39% of wealth at the risk-free rate to invest a total of 139% in short-term nominal bonds portfolios. Finally, we see less large quantities of short-sales $(1 - \alpha)$ or, in some cases, no short-sales for the risk-free Treasury bill, for the same degree of risk aversion as the investment horizon increases. For example, an investor with $\gamma = 20$ goes short in the risk-free bond at the monthly frequency but goes long in both long-term nominal bonds and the risk-free bond at longer investment horizons. The same situation occurs with long-term bonds with respect to short-term ones in the sense that we see less large quantities for a

Table 1: Descriptive Statistics for the portfolio measures of liquidity and bond returns

| | Short-term | | | | Long-term | | |
|---|---|---|---|---|---|---|---|
| | $L^{z\text{-}asw}_{10,t}$ | $R^{N}_{t+1}$ | $R^{TIPS}_{t+1}$ | | $L^{z\text{-}asw}_{20,t}$ | $R^{N}_{t+1}$ | $R^{TIPS}_{t+1}$ |
| Panel A: Monthly frequency | | | | | | | |
| Mean | 0.00 | 1.04 | 1.02 | | 0.00 | 1.06 | 1.03 |
| Stdev | 1.00 | 0.02 | 0.02 | | 1.00 | 0.04 | 0.03 |
| Skewness | 2.58 | 0.03 | -0.34 | | 1.91 | 0.49 | 0.10 |
| Kurtosis | 11.15 | 3.70 | 6.30 | | 8.36 | 5.60 | 5.91 |
| Percentiles | | | | | | | |
| 5% | -0.95 | 1.01 | 0.99 | | -1.17 | 0.99 | 0.98 |
| 50% | -0.19 | 1.05 | 1.02 | | -0.18 | 1.06 | 1.03 |
| 95% | 2.15 | 1.07 | 1.05 | | 2.06 | 1.11 | 1.07 |
| Cross correlations | | | | | | | |
| $\Delta_{n,t}$ | 1.00 | | | | 1.00 | | |
| $R^{N}_{t+1}$ | 0.05 | 1.00 | | | -0.13 | 1.00 | |
| $R^{TIPS}_{t+1}$ | 0.33 | 0.46 | 1.00 | | 0.18 | 0.59 | 1.00 |
| Auto correlations | | | | | | | |
| 1-day | 0.99 | 0.95 | 0.96 | | 0.99 | 0.95 | 0.94 |
| 2-day | 0.98 | 0.91 | 0.92 | | 0.98 | 0.90 | 0.89 |
| 5-day | 0.95 | 0.80 | 0.78 | | 0.95 | 0.77 | 0.72 |
| 22-day | 0.76 | 0.07 | 0.06 | | 0.77 | -0.06 | -0.11 |
| Unit root test | | | | | | | |
| DF p-value | 0.02 | 0.01 | 0.01 | | 0.14 | 0.01 | 0.01 |
| Panel B: Quarterly frequency | | | | | | | |
| Mean | 0.00 | 1.05 | 1.02 | | 0.00 | 1.06 | 1.03 |
| Stdev | 1.00 | 0.03 | 0.03 | | 1.00 | 0.07 | 0.05 |
| Skewness | 2.58 | 0.04 | -0.55 | | 1.91 | 0.28 | -0.26 |
| Kurtosis | 11.15 | 2.80 | 6.78 | | 8.36 | 3.32 | 4.27 |
| Percentiles | | | | | | | |
| 5% | -0.95 | 1.00 | 0.98 | | -1.17 | 0.95 | 0.95 |
| 50% | -0.19 | 1.04 | 1.02 | | -0.18 | 1.06 | 1.04 |
| 95% | 2.15 | 1.09 | 1.07 | | 2.06 | 1.17 | 1.11 |
| Cross correlations | | | | | | | |
| $\Delta_{n,t}$ | 1.00 | | | 1.00 | | | |
| $R^{N}_{t+1}$ | -0.14 | 1.00 | | | -0.23 | 1.00 | |
| $R^{TIPS}_{t+1}$ | 0.37 | 0.28 | 1.00 | | 0.23 | 0.59 | 1.00 |
| Auto correlations | | | | | | | |
| 1-day | 0.99 | 0.98 | 0.99 | | 0.99 | 0.98 | 0.98 |
| 2-day | 0.98 | 0.96 | 0.97 | | 0.98 | 0.96 | 0.95 |
| 5-day | 0.95 | 0.92 | 0.93 | | 0.95 | 0.91 | 0.89 |
| 22-day | 0.90 | 0.86 | 0.86 | | 0.90 | 0.85 | 0.81 |
| Unit root test | | | | | | | |
| DF p-value | 0.02 | 0.01 | 0.01 | | 0.14 | 0.01 | 0.01 |

portfolio of long-term vs short-term bonds. This indicates that a smaller portion of the portfolio is devoted to risky assets as investment horizons increase or when long-run assets are available.

Results for CRRA preferences are very similar to those for MV. In theory, what differentiates a Mean-variance investor from a CRRA investor is that the latter has a preference for higher order moments and not only for the expected return and its variance, thus their risky position depends on relative risk aversion. However, empirical results in Table 2, show that investors seem not to be primarily affected in their decisions by the first two return moments. So, the effect of higher order moments of CRRA investors seem not to be strong enough, especially for TIPS. The biggest holding difference is for short-term nominal bonds at the monthly frequency, where CRRA investors with

*Continuation*: Descriptive Statistics

| | Short-term | | | Long-term | | |
|---|---|---|---|---|---|---|
| | $L^{z\text{-}asw}_{10,t}$ | $R^N_{t+1}$ | $R^{TIPS}_{t+1}$ | $L^{z\text{-}asw}_{20,t}$ | $R^N_{t+1}$ | $R^{TIPS}_{t+1}$ |
| Panel A: Annual frequency | | | | | | |
| Mean | 0.00 | 1.06 | 1.04 | 0.00 | 1.10 | 1.06 |
| Stdev | 1.00 | 0.04 | 0.05 | 1.00 | 0.09 | 0.08 |
| Skewness | 2.58 | -0.14 | 0.02 | 1.91 | 0.16 | 0.06 |
| Kurtosis | 11.15 | 2.33 | 3.06 | 8.36 | 3.70 | 2.76 |
| Percentiles | | | | | | |
| 5% | -0.95 | 0.99 | 0.96 | -1.17 | 0.94 | 0.93 |
| 50% | -0.19 | 1.06 | 1.04 | -0.18 | 1.09 | 1.07 |
| 95% | 2.15 | 1.12 | 1.11 | 2.06 | 1.29 | 1.21 |
| Cross correlations | | | | | | |
| $\Delta_{n,t}$ | 1.00 | | | 1.00 | | |
| $R^N_{t+1}$ | -0.50 | 1.00 | | -0.60 | 1.00 | |
| $R^{TIPS}_{t+1}$ | 0.36 | -0.04 | 1.00 | 0.00 | 0.46 | 1.00 |
| Auto correlations | | | | | | |
| 1-day | 0.99 | 0.99 | 1.00 | 0.99 | 0.99 | 0.99 |
| 2-day | 0.98 | 0.98 | 0.99 | 0.98 | 0.98 | 0.98 |
| 5-day | 0.95 | 0.96 | 0.97 | 0.95 | 0.95 | 0.95 |
| 22-day | 0.76 | 0.83 | 0.86 | 0.77 | 0.78 | 0.82 |
| Unit root test | | | | | | |
| DF p-value | 0.02 | 0.23 | 0.09 | 0.14 | 0.05 | 0.02 |

The liquidity measure corresponds to the TIPS Liquidity proposed by Christensen and Gillan (2011). U.S. daily data from January 1, 2004 to December 30 2012 in basis points.

different levels of risk aversion tend to hold larger quantities.

There are important differences in the optimal portfolio weights between short-term and long-term nominal bonds with both types of preferences. In fact, equally risk-averse investors tend to hold bigger positions on short-term bonds relative to long-term ones, i.e. the short-term bond weight typically exceeds the long-term weight for the same kind of bond. However, these differences become smaller when the investment horizon become longer. Bonds with a longer maturity will usually pay a higher interest rate than shorter-term bonds. However, long-term bonds have greater duration than short-term bonds, so interest rate changes will have a greater effect on long-term bonds than on short-term bonds. As a result, investors are more conservative holding smaller positions in long-term bonds relative to short-term bonds, given that they would offer greater stability and lower risk.

Investors also hold bigger positions in nominal bonds relative to TIPS bonds. These differences could be attributed, at least in the case of CRRA investor, to the negative skewness in short-term TIPS bond returns for monthly and quarterly frequency, as Table 1 shows. Investors prefer positive skewness, because it implies a low probability of obtaining a large negative return. Then, investors tend to the extreme portfolios (Sharpe ratio driven, skewness driven or kurtosis driven) and avoid being stuck in the middle.

## Table 2: Unconditional Portfolio Weights

| | Mean-Variance investor | | | | Power Utility investor | | | |
|---|---|---|---|---|---|---|---|---|
| | Treasury | | TIPS | | Treasury | | TIPS | |
| $\gamma$ | Short-term | Long-term | Short-term | Long-term | Short-term | Long-term | Short-term | Long-term |
| | | | | Monthly frequency | | | | |
| 2 | 79.75 | 15.58 | 33.53 | 12.32 | 139.32 | 10.15 | 7.26 | 6.57 |
| | [29.70] | [3.48] | [5.73] | [1.80] | [5.91] | [0.77] | [1.16] | [0.45] |
| 5 | 31.90 | 6.23 | 13.41 | 4.93 | 64.58 | 5.11 | 7.26 | 3.63 |
| | [11.88] | [1.39] | [2.29] | [0.72] | [10.65] | [0.65] | [1.16] | [0.59] |
| 10 | 15.95 | 3.12 | 6.71 | 2.46 | 30.15 | 2.79 | 3.97 | 1.95 |
| | [5.94] | [0.70] | [1.15] | [0.36] | [6.50] | [0.41] | [0.72] | [0.35] |
| 20 | 7.98 | 1.56 | 3.35 | 1.23 | 14.51 | 1.43 | 2.06 | 1.00 |
| | [2.97] | [0.35] | [0.57] | [0.18] | [3.60] | [0.22] | [0.39] | [0.19] |
| | | | | Quarterly frequency | | | | |
| 2 | 29.45 | 6.54 | 10.11 | 5.31 | 29.35 | 6.94 | 6.00 | 4.00 |
| | [6.30] | [0.81] | [1.93] | [0.80] | [1.57] | [0.64] | [0.92] | [0.58] |
| 5 | 11.78 | 2.61 | 4.05 | 2.12 | 14.04 | 3.01 | 2.88 | 1.82 |
| | [2.52] | [0.33] | [0.77] | [0.32] | [1.45] | [0.36] | [0.60] | [0.35] |
| 10 | 5.89 | 1.31 | 2.02 | 1.06 | 6.98 | 1.50 | 1.51 | 0.94 |
| | [1.26] | [0.16] | [0.39] | [0.16] | [0.77] | [0.18] | [0.33] | [0.19] |
| 20 | 2.95 | 0.65 | 1.01 | 0.53 | 3.44 | 0.75 | 0.77 | 0.47 |
| | [0.63] | [0.08] | [0.19] | [0.08] | [0.39] | [0.09] | [0.17] | [0.09] |
| | | | | Annual frequency | | | | |
| 2 | 13.91 | 4.71 | 3.45 | 3.23 | 14.78 | 3.62 | 3.45 | 3.20 |
| | [1.60] | [0.64] | [0.73] | [0.37] | [1.34] | [0.39] | [0.84] | [0.49] |
| 5 | 5.57 | 1.88 | 1.38 | 1.29 | 6.30 | 1.77 | 1.41 | 1.35 |
| | [0.64] | [0.26] | [0.29] | [0.15] | [0.73] | [0.28] | [0.36] | [0.23] |
| 10 | 2.78 | 0.94 | 0.69 | 0.65 | 3.11 | 0.92 | 0.71 | 0.68 |
| | [0.32] | [0.13] | [0.15] | [0.07] | [0.37] | [0.15] | [0.18] | [0.12] |
| 20 | 1.39 | 0.47 | 0.35 | 0.32 | 1.54 | 0.46 | 0.35 | 0.34 |
| | [0.16] | [0.06] | [0.07] | [0.04] | [0.18] | [0.08] | [0.09] | [0.06] |

This table shows estimates of the optimal unconditional portfolio choice of investors. This is computed by applying a standard GMM procedure to the unconditional euler equation (9). Each panel shows a different investment horizon: monthly, quarterly and annual. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. Weights in the table correspond to the risky asset. In brackets are the Newey-West (12 lags) standard errors. I used U.S. data from January 1, 2004 to December 30 2011.

## 4.2 Conditional portfolio weights

### 4.2.1 Non-parametric optimal portfolio function

In this section I present the optimal portfolio weights as function of the liquidity differential between inflation-indexed bonds and nominal bonds (liquidity premium), represented by $Z_t$. I apply the utility maximization framework presented above with respect to $Z_t$. For each kernel grid point,[6] I optimize the portfolio weight by maximizing the representative agent's marginal utility in that state using a GMM inference technique. The portfolio weights that follow from the optimization of the expected utility under MV and CRRA preferences are presented in this section.

Table 3 shows estimates of the optimal conditional portfolio choice of investors (Weight) and their corresponding standard errors (Std) obtained by applying the Politis and Romano (1994) bootstrap procedure. I use this stationary bootstrap procedure to preserve autocorrelation properties of the data in the bootstrap samples.[7] The standard errors are presented only in order to assess the precision of the non-parametric method used. Each panel shows a different investment horizon (monthly, quarterly and annual), and they present the portfolio allocation problems considered before: two, where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset, and another two where the investor chooses between a portfolio of short-term or long-term TIPS and a risk-free asset, with each of them considering a MV and a CRRA investor.

Figure 2 are the companion graphs to Table 3. Each figure shows the optimal portfolio weight as a function of liquidity $\alpha(Z_t)$ represented by the bold line. Additionally, in each figure the thinner horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram of liquidity premium (scaled to add up to 30). Results presented in Table 3 and in Figure 2 correspond to the case when the coefficient of relative risk aversion is equal to $\gamma = 20$. A number of results emerge from this analysis. First, the liquidity premium seems to be a significant determinant of the portfolio allocation to U.S. government bonds. For instance, for a MV investor and at the monthly horizon, liquidity is a strong determinant of the allocation to short-term and long-term nominal bonds, with the optimal weight ranging from 9.41 at Liquidity= (-1) to 2.31 at Liquidity=5, as Table 3 shows. This indicates that an increase in the liquidity premium (i.e., liquidity conditions worsen) is accompanied by a strong decrease in the optimal allocation in short-term nominal bonds.

I have a similar result for the long-term nominal bonds with weights ranging from 2.05 to 0.27. Furthermore, liquidity also seems to be an important determinant of the allocation to TIPS. In this case, an increase in liquidity premium produces a decrease in the optimal allocation to both short-term, and long-term TIPS bonds. However, the effect is less strong with weights ranging from 3.80 to 3.16 for short-term, and from 1.31 to 0.65 for long-term for liquidity ranging between -1 and 5, respectively.

At quarterly and annual frequencies, optimal allocation still responds to changes in liquidity but mainly at high levels of liquidity premium. What we see is that the conditional weight is very close to

[6] I define fifteen not evenly spaced realizations of the liquidity ranging from its mean minus one standard deviation to its means plus three standard deviations, which correspond to the interior 95% of the empirical distribution of the liquidity premium. Alternatively, I also define fifteen not evenly spaced realizations of the liquidity ranging between its minimum and maximum value, however results are broadly the same with both grids.

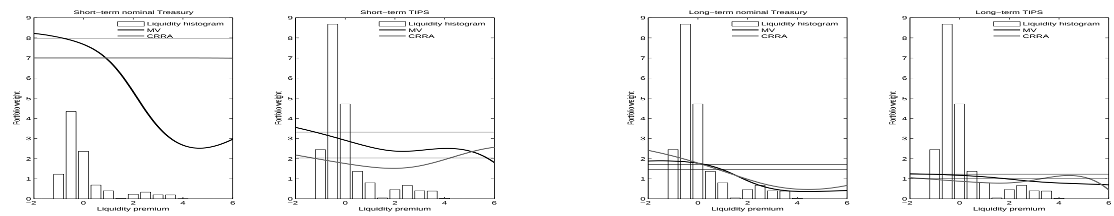
[7] This method is a variation of the standard block bootstrap that manages to create bootstrap series that are strictly stationary which accounts for the autocorrelation in the data.

the unconditional weight for low levels of liquidity (i.e. liquidity= -1 to 2), however optimal allocation starts to respond to changes in the liquidity when market liquidity conditions worsen (i.e. liquidity > 2). Interestingly, the investor tends to substitute cash for nominal bonds, and TIPS bonds for cash when the liquidity rises above its mean plus about 4 standard deviations, as Figure 2 shows.

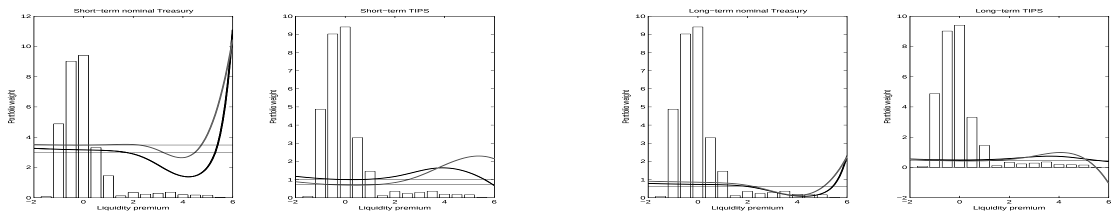
Second, conditional allocations in risky assets decrease as liquidity conditions worsen. In particular, an increase in the liquidity differential between nominal and TIPS bonds lead to: lower optimal portfolio allocations on nominal Treasury bonds, and also lower optimal portfolio allocations in TIPS, but at different levels of liquidity. When the liquidity premium is low (i.e. the liquidity differential between nominal and TIPS bonds is small), we see that the optimal allocation to either nominal or TIPS bonds is mostly unresponsive to liquidity premium, and it is very close to unconditional allocation. This occurs in the negative range of liquidity and also in the center of the distribution.

Figure 2: Optimal portfolio weights as a function of 10-year liquidity premium

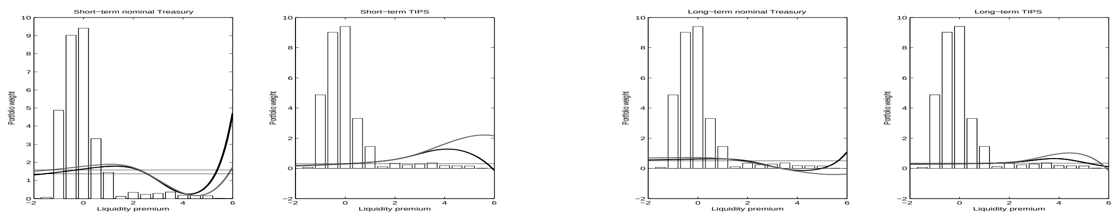
(a) Monthly frequency




(b) Quarterly frequency




(c) Annual frequency




In each panel the thinner horizontal lines represents the optimal unconditional allocation. The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The bold line represents the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. In the first row, both the investment horizon and the rebalancing frequency are one-month; in the second row, one-quarter; and in the third, one-year.

When the liquidity premium is high (i.e. in presence of big liquidity differentials between nominal

Table 3: Conditional Portfolio Weights ($\gamma = 20$)

| | Mean-Variance investor | | | | | | | | Power Utility investor | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Treasury | | | | TIPS | | | | Treasury | | | | TIPS | | | |
| | Short-term | | Long-term | | Short-term | | Long-term | | Short-term | | Long-term | | Short-term | | Long-term | |
| $Z$ | Weight | Stdev | Weight | Stdev | Weight | Stdev | Weight | Stdev | Weight | Stdev | Weight | Stdev | Weight | Stdev | Weight | Stdev |
| Monthly frequency | | | | | | | | | | | | | | | | |
| -1 | 9.41 | 0.72 | 2.05 | 0.17 | 3.80 | 0.50 | 1.31 | 0.19 | 7.00 | 0.00 | 2.21 | 0.26 | 2.13 | 0.52 | 0.99 | 0.18 |
| 0 | 9.07 | 0.72 | 2.02 | 0.17 | 3.68 | 0.55 | 1.33 | 0.19 | 7.00 | 0.00 | 2.10 | 0.24 | 2.01 | 0.52 | 0.98 | 0.18 |
| 2 | 7.06 | 0.93 | 1.42 | 0.20 | 3.61 | 0.66 | 1.38 | 0.21 | 7.00 | 0.00 | 1.51 | 0.23 | 1.90 | 0.57 | 1.03 | 0.18 |
| 4 | 2.99 | 0.95 | 0.41 | 0.17 | 3.15 | 0.50 | 0.82 | 0.20 | 7.00 | 0.00 | 0.55 | 0.32 | 2.56 | 0.77 | 1.19 | 0.26 |
| 5 | 2.31 | NaN | 0.27 | NaN | 3.16 | NaN | 0.65 | NaN | 7.00 | 0.00 | 0.33 | 0.42 | 3.33 | 1.06 | 1.12 | 0.43 |
| Quarterly frequency | | | | | | | | | | | | | | | | |
| -1 | 1.46 | 0.16 | 0.75 | 0.07 | 0.24 | 0.07 | 0.51 | 0.08 | 3.48 | 0.36 | 0.88 | 0.10 | 0.74 | 0.15 | 0.46 | 0.08 |
| 0 | 1.63 | 0.16 | 0.73 | 0.07 | 0.30 | 0.08 | 0.49 | 0.09 | 3.48 | 0.36 | 0.85 | 0.09 | 0.71 | 0.16 | 0.43 | 0.08 |
| 2 | 1.68 | 0.21 | 0.61 | 0.08 | 0.60 | 0.10 | 0.58 | 0.11 | 3.51 | 0.39 | 0.68 | 0.08 | 0.79 | 0.16 | 0.49 | 0.09 |
| 4 | 0.24 | 0.21 | 0.07 | 0.11 | 1.41 | 0.19 | 0.76 | 0.21 | 2.43 | 1.05 | 0.08 | 0.14 | 1.38 | 0.28 | 0.84 | 0.17 |
| 5 | -0.21 | NaN | -0.03 | NaN | 1.65 | NaN | 0.77 | NaN | 2.15 | 1.44 | -0.03 | 0.19 | 1.88 | 0.47 | 1.04 | 0.27 |
| Annual frequency | | | | | | | | | | | | | | | | |
| -1 | 0.24 | 0.08 | 0.59 | 0.06 | 0.24 | 0.08 | 0.29 | 0.04 | 1.63 | 0.17 | 0.71 | 0.07 | 0.23 | 0.08 | 0.31 | 0.05 |
| 0 | 0.30 | 0.08 | 0.62 | 0.06 | 0.30 | 0.09 | 0.30 | 0.05 | 1.78 | 0.17 | 0.72 | 0.07 | 0.30 | 0.08 | 0.32 | 0.05 |
| 2 | 0.60 | 0.09 | 0.50 | 0.08 | 0.60 | 0.11 | 0.39 | 0.06 | 1.72 | 0.17 | 0.42 | 0.08 | 0.52 | 0.09 | 0.38 | 0.06 |
| 4 | 1.41 | 0.21 | -0.18 | 0.08 | 1.41 | 0.22 | 0.68 | 0.12 | 0.26 | 0.31 | -0.18 | 0.07 | 1.20 | 0.16 | 0.76 | 0.12 |
| 5 | 1.65 | NaN | -0.32 | NaN | 1.65 | NaN | 0.67 | NaN | -0.23 | 0.42 | -0.35 | 0.10 | 1.71 | 0.25 | 1.08 | 0.20 |

This table shows estimates of the optimal conditional portfolio choice of investors. This is computed by applying a standard GMM procedure to the conditional euler equation (8). Each panel shows a different investment horizon: monthly, quarterly and annual. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. Weights correspond to the risky asset. Standard errors (Std) are obtained applying the Politis and Romano (1994) bootstrap procedure. I used U.S. data from January 1, 2004 to December 30 2011.

and TIPS bonds), portfolio allocation on both nominal bonds and TIPS bonds decreases. However, this occurs at different levels of liquidity. In particular, the investor starts to decrease their position in nominal bonds at liquidity=2, but when there is insufficient liquidity, the investor holds a larger position in nominal bonds. On the other hand, portfolio allocation on TIPS bonds behaves in the reverse direction. That is, the investor only decreases asset allocation to TIPS in the upper positive part of liquidity (i.e. when the liquidity premium is very high), while between liquidity=2 and liquidity=4 TIPS bonds allocations increases, being above the unconditional value. Thus, in general, portfolio allocation for each type of bonds (nominal and TIPS) moves in cycles and each of them has its own cycle. Typically, when one type of bond is performing well, the other may not be performing as well in terms of liquidity, and the allocation rule reflects this situation.

Third, I find in general that the shape of the optimal portfolio policy functions of mean-variance and CRRA investors, with the same degree of risk aversion, are similar even though they have different levels. This suggest that investors seems to be primarily affected in their decisions by the first two return moments. Thus, the effect of higher order moments of CRRA investors exist but it seems not to be strong enough. However, this is not true at the monthly frequency. In this case, portfolio policies differ substantially which can be attributed to time variation in the higher order moments of the return distribution. This result is not induced by the choice of the kernel bandwidth, given that I explicitly control for it by constraining the kernel to be the same for the mean-variance and the CRRA preferences.[8]

Fourth, the effect of liquidity is a decreasing function of the investment horizon. For a given degree of risk aversion, the size of the optimal portfolio weight differs considerably across investment horizons. I find that as investment horizons became longer, the smaller the optimal portfolio weight, and the less that is invested in the risky asset. In particular, for the same degree of risk aversion investors react less abruptly to an increase in the liquidity premium when the investment horizon is one-year, than when the investment horizon is one-month.

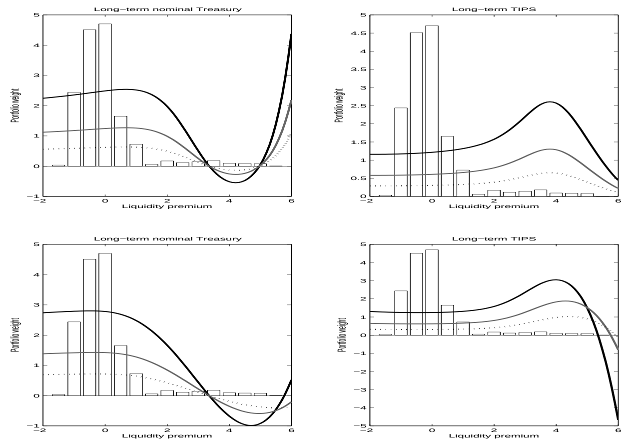
For instance, we can see from Table 3 that when liquidity is equal to its mean ($Z_t = 0$) a MV investor with $\gamma = 20$ reduces the cash holdings from 2.02 to 0.62 when the investment horizon increases from one-month to one-year. This means that the investor borrows 102% of wealth at the risk-free rate to invest a total of 202% in short-term nominal bonds when the investment horizon is one-month. However, when the investment horizon becomes larger, the investor takes a long position in both assets holding 62% of their wealth in short-term nominal bonds and 38% in cash. The same occurs when I consider a CRRA investor. For example, considering the same case, but for long-term TIPS bonds, a CRRA investor reduces their bonds positions from 98% to 32%, as Table 3 shows.

Fifth, different degrees of risk aversion mainly change the level of the portfolio function but have

[8] Non-parametric methods are typically indexed by a bandwidth or tuning parameter which controls the degree of complexity. The choice of bandwidth is often critical to implementation. In this application, the bandwidth is given by: $h = \lambda\sigma_z T^{-1/K+4}$, where $K = 1$ which is the dimension of $Z$ (I am considering only one predictor variable which is liquidity), $\sigma(Z)$ is the standard deviation of the predictor variable, $T = 2086$ is the sample size and $\lambda$ is a constant. For a big enough value of $\lambda$, I obtain a flat portfolio weight and small $\lambda$ produce a very noise portfolio weight function. I consider values ranging from 9 to 3 for $\lambda$. These values guarantee bigger weight to an observation located at the mean of liquidity variable (which is zero), smaller weights to observations located one standard deviation away from the mean ($Z_t = \pm 1$), and even smaller weights to observations located two standard deviation away from the mean, etc. The results presented in this section correspond to $\lambda = 6$.

little impact on the shape of this function, as is shown in Figure 3. In this figure, I only plot the portfolio policies for the long-term nominal (left column) and TIPS bonds (right column) for a one-year investment horizon. The first row in the figure corresponds to a mean-variance investor, and the second row to a CRRA investor. Finally, in each panel bold black lines represent an investor with $\gamma = 5$, the bold grey line with $\gamma = 10$ and the dotted line with $\gamma = 20$. Looking at Figure 3, we see that the more risk-averse the investor becomes, the smaller the optimal portfolio weight, so the less that is invested in the risky asset. Furthermore, more risk-averse investors react less abruptly to an increase in the liquidity premium.

Figure 3: Optimal portfolio weights as a function of 10-year liquidity premium (Mean-variance and CRRA investor with different values for $\gamma$)




The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The lines represent the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. Bold black line represent an investor with $\gamma = 5$, the bold grey line for $\gamma = 10$ and dotted line for $\gamma = 20$. In the first row correspond to the case of mean-variance investor and the second row to the CRRA investor. The investment horizon and the rebalancing frequency in this figure correspond to one-year.

To summarize, and in general, results consistently show that the optimal allocation to short-term or long-term bonds is mostly unresponsive to changes in liquidity conditions at low levels (i.e. at liquidity= -1 to 4). However once liquidity reaches certain levels (liquidity $> 4$), which indicates that market liquidity conditions have worsened, then the investor starts to respond by decreasing the positions in TIPS and increasing the position in nominal bonds.

Additionally, the above conclusion is not determined by the level of risk aversion, the investment horizon or the investor preferences. The relation between optimal portfolio weights and liquidity premium remains the same for different values of risk-aversion, different investment horizons and also across investors' preferences. The characteristics mainly change the level of the portfolio function that have a small impact on the function shape, except for the monthly frequency.

#### 4.2.2 Do weights really respond to changes in liquidity?

The main question of this paper is whether or not the weights respond to changes in liquidity. To test whether or not a portfolio weight is statistically different from zero is pointless in this context, simply because it does not provide an answer for the question asked above. What I do next, following Ghysels and Pereira (2008), is to statistically test this question by using the following approximation:

$$H_0 : \frac{\partial \alpha(Z)}{\partial Z} |_{Z=\bar{Z}} \quad \cong \frac{\alpha(\bar{Z}+0.1)-\alpha(\bar{Z}-0.1)}{0.2} = 0 \tag{8}$$

where the first derivative of $\alpha(Z)$ is approximated by a finite difference which allows me to compute the slope of the optimal portfolio weight function at each value of the predictor variable.

Table 4 shows the point estimate slopes and t-stat computed using the standard errors obtained also from the Politis and Romano (1994) stationary bootstrap procedure. I draw one main conclusion from this table which is consistent with the results presented above. It is clear that optimal portfolio policy is not linear or constant in liquidity. For the two investor preferences the short-term nominal and the TIPS bonds portfolio policy responds to changes in liquidity. This conclusion is derived from the fact that the null hypothesis is rejected indicating that all slopes are statistically significant at the 10% level or less. The only case where slopes are not statistically significant is for short-term TIPS bonds with MV preferences. The other case where we can not reject the null hypothesis is for short-term nominal bonds with CRRA preferences. In this case, the optimal portfolio function is constant but smaller than the unconditional weight.

For long-term TIPS, $\alpha(Z_t)$ is almost constant and statistically not different from zero over the negative range of liquidity until $Z_t = 2$. After that the slopes are positive and over the last range of liquidity they are negative and statistically significant. I find the same results for both investor preferences. The optimal portfolio function for long-term nominal bonds goes in the opposite way. It starts by being flat and statistically not different from zero, then slopes become negative, and over the the end range of liquidity, slopes are positive and statistically significant.

Overall, I can conclude that optimal portfolio choice is unresponsive over the negative and first positive range of liquidity, however portfolio allocations start to react as liquidity conditions worsen. This conclusion regarding the general shape of the portfolio weight functions is reliable in the sense that non-parametric techniques used here produce a consistent estimator of the portfolio functions.

### 4.3 Does a conditional allocation strategy imply improved asset allocation and performance?

From the standpoint of practical advice to portfolio investors, an additional natural question to ask is whether or not to follow a conditional strategy translates into improved out-of-sample asset allocation and performance. The idea is that at the start of each period (one-month, one-quarter or one-year), one investor makes portfolio allocations conditional upon observing a particular liquidity signal (conditional strategy). I compare his/her performance to that of another investor who ignores any change in liquidity in making his/her portfolios allocation choices (unconditional strategy).

Table 4: Point estimates for the slope of the conditional portfolio weight function

| | Mean-Variance investor | | | | | | | | Power Utility investor | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Treasury | | | | TIPS | | | | Treasury | | | | TIPS | | | |
| | Short-term | | Long-term | | Short-term | | Long-term | | Short-term | | Long-term | | Short-term | | Long-term | |
| $Z$ | Slope | t-stat | Slope | t-stat | Slope | t-stat | Slope | t-stat | Slope | t-stat | Slope | t-stat | Slope | t-stat | Slope | t-stat |
| | | | | | | | | Monthly frequency | | | | | | | | |
| -1 | -0.30 | -1.62 | -0.01 | -0.31 | -0.15 | -1.14 | 0.01 | 0.18 | 0.00 | -0.03 | -0.08 | -1.49 | -0.14 | -2.02 | -0.01 | -0.53 |
| 0 | -0.40 | -2.19 | -0.06 | -1.10 | -0.08 | -0.58 | 0.03 | 0.76 | 0.00 | -0.26 | -0.15 | -2.39 | -0.11 | -1.63 | 0.00 | -0.05 |
| 2 | -2.08 | -3.33 | -0.67 | -4.76 | -0.07 | -0.35 | -0.07 | -0.62 | 0.00 | 0.07 | -0.47 | -3.96 | 0.05 | 0.58 | 0.06 | 1.42 |
| 4 | -1.01 | -2.01 | -0.19 | -1.64 | -0.14 | -0.52 | -0.24 | -2.75 | 0.00 | 0.61 | -0.31 | -2.06 | 0.66 | 1.68 | -0.01 | -0.03 |
| 4.5 | -0.71 | -1.67 | -0.14 | -1.27 | -0.03 | -0.05 | -0.18 | -2.51 | 0.00 | -1.76 | -0.23 | -1.62 | 0.76 | 1.77 | -0.07 | -0.31 |
| 6.0 | 2.43 | 4.31 | -0.10 | -1.30 | -0.01 | -0.05 | -0.13 | -2.35 | 0.00 | 0.65 | 0.42 | 0.93 | 0.91 | 2.24 | -0.06 | -0.28 |
| | | | | | | | | Quarterly frequency | | | | | | | | |
| -1 | 0.17 | 4.26 | -0.03 | NaN | 0.06 | 3.08 | -0.03 | -1.60 | -0.02 | -0.24 | -0.03 | -1.08 | -0.05 | -2.43 | -0.03 | -1.06 |
| 0 | 0.18 | 2.81 | -0.02 | -1.00 | 0.08 | 3.95 | 0.00 | -0.05 | 0.01 | 0.10 | -0.03 | -1.04 | -0.02 | -1.10 | -0.01 | -0.72 |
| 2 | -0.35 | -2.85 | -0.18 | -4.56 | 0.28 | 6.51 | 0.10 | 2.42 | -0.08 | -0.33 | -0.23 | -3.31 | 0.13 | 3.78 | 0.08 | 3.26 |
| 4 | -0.62 | -3.18 | -0.17 | -1.98 | 0.31 | 2.52 | 0.01 | 0.04 | -0.55 | -1.05 | -0.18 | -2.29 | 0.46 | 2.21 | 0.24 | 1.84 |
| 4.5 | -0.53 | -2.82 | -0.13 | -1.61 | 0.27 | 2.18 | 0.00 | 0.03 | -0.44 | -0.84 | -0.14 | -1.79 | 0.48 | 2.18 | 0.23 | 1.71 |
| 6.0 | 0.93 | 2.76 | 0.65 | 1.65 | -0.31 | 1.96 | -0.05 | -0.91 | 0.57 | 0.54 | 0.45 | 1.68 | 0.53 | 2.07 | -0.43 | -1.08 |
| | | | | | | | | Annual frequency | | | | | | | | |
| -1 | 0.06 | 3.23 | 0.03 | 3.00 | 0.06 | 2.78 | 0.01 | 0.66 | 0.14 | 5.12 | 0.02 | 1.12 | 0.05 | 3.06 | 0.00 | -0.15 |
| 0 | 0.08 | 4.26 | 0.03 | 1.48 | 0.08 | 3.39 | 0.02 | 1.46 | 0.15 | 3.41 | -0.02 | -0.61 | 0.07 | 3.74 | 0.01 | 0.64 |
| 2 | 0.28 | 6.16 | -0.26 | -4.02 | 0.28 | 5.94 | 0.10 | 4.41 | -0.45 | -3.13 | -0.29 | -5.80 | 0.19 | 5.90 | 0.08 | 3.34 |
| 4 | 0.31 | 1.83 | -0.21 | -3.64 | 0.31 | 2.04 | 0.04 | 0.54 | -0.65 | -3.75 | -0.24 | -4.48 | 0.48 | 3.96 | 0.30 | 2.78 |
| 4.5 | 0.24 | 1.26 | -0.15 | -2.61 | 0.24 | 1.54 | -0.01 | -0.21 | -0.52 | -3.04 | -0.18 | -3.45 | 0.50 | 3.66 | 0.31 | 2.55 |
| 6.0 | 0.87 | 1.45 | 0.54 | 2.34 | -0.33 | 1.98 | -0.11 | -0.76 | 0.37 | 1.92 | 0.06 | 1.57 | 0.12 | 3.25 | -0.34 | -2.48 |

This table shows the point estimates slopes and their standard errors obtained from Politis and Romano (1994) stationary bootstrap procedure. This is computed by approximating the first derivative of $\alpha(Z)$ by equation (10). Each panel shows a different investment horizon: monthly, quarterly and annual. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. I used U.S. data from January 1, 2004 to December 30 2011.

I used rolling estimation approach, which consists of estimating a series of out-of-sample portfolio returns by using a rolling estimation window over the entire data set. Specifically, I choose an estimation window of length $M$=260 days (1 year). In each day, starting from $t = M + 1$, I use the data in the previous $M$ days to estimate the optimal portfolio weights. In other words, each investor has an investment horizon of one-year and uses all data available until period $T - M$ to choose his/her first portfolio weights. Next, I use those weights to compute the portfolio returns. Repeating this procedure, involve adding the information for the next period in the data set and dropping the earliest period (keeping the window length fixed), until the end of the data set is reached. In this way, I obtain a time series of portfolio returns for each (unconditional and conditional) strategy.

To compute out-of-sample performance of this two different strategies, I compute the out-of-sample Sharpe ratio of strategy $j$, defined as the sample mean of out-of-sample excess returns (over the risk-free asset), $\mu_j$, divided by their sample standard deviation, $\sigma_j$, for strategy $j = U, C$

$$SR_j = \frac{\mu_j}{\sigma_j}. \tag{9}$$

In addition, I calculate the certainty equivalent rates of return ($CER$) for each strategy to judge its relative performance. The $CER$ represents the risk-free rate of return that investor is willing to accept instead of undertaking the risky portfolio strategy. Formally, I compute the $CER$ of strategy $j$

$$CER_j = \mu_j - \frac{\gamma}{2}\sigma_j^2, \tag{10}$$

where $\mu_j$ and $\sigma_j^2$ are the mean and variance of out-of-sample excess returns for strategy $j = U, C$. To test whether or not the Sharpe ratios, and the certainty equivalent returns of two strategies are statistically distinguishable, I test the following null hypothesis $Ho : SR_U - SR_C$ and $Ho : CER_U - CER_C$. This difference represents the gain (or loss) in returns from investing in unconditional strategy versus conditional strategy. I compute the p-value of the differences by using the Politis and Romano (1994) stationary bootstrap procedure ($pv - boot$).[9] Finally, an useful benchmark are the in-sample Sharpe ratios and the certainty equivalent returns (to assess the effect of estimation error), calculated for the different portfolio strategies by using the entire time series of excess returns.

Table 6 shows results assuming both investors are mean-variance optimizer with a one-year investment horizon, and $\gamma = 10$. Panel A shows the $CER$ and the $SR$ calculated with the entire data set (in-sample analysis). The in-sample Sharpe ratios are all positive (except for short-term nominal bonds), being the performing of the conditional strategy better than the unconditional strategy for all portfolios. For instance, for a nominal long-term portfolio the Sharpe ratio of unconditional strategy is equal to 0.12 versus 0.36 of the conditional strategy, indicating that with the conditional strategy the investor takes on less risk to achieve the same return. For the same portfolio, the $CER_U$ is equal to 0.53 vs 0.60 of the $CER_C$. This means that an investor requires a higher risk-free return to give up the opportunity to invest in the portfolio following a conditional strategy.

[9] I replicate the process described in Appendix A 1000 times. For each such replication, I compute the optimal allocations for each investor through one year (260 days). At every point in time, the investors are allowed to utilize just the information available up to that point in time. I calculate the difference in certainty equivalent between the two strategies and the adjusted Sharpe ratio for each replication. Finally, I count the proportion of times in 1000 replications that these differences exceed the certainty equivalent and adjusted Sharpe ratio based on the original data for a given set of results.

Table 5: Sharpe ratios and certainty equivalent returns (Mean-Variance investor with $\gamma = 10$)

Panel A: In- sample results

| | | Unconditional | | Conditional | | Differential | | $Ho$:differential=0 | |
|---|---|---|---|---|---|---|---|---|---|
| | | $SR$ | $CER$ | $SR$ | $CER$ | $SR_U - SR_C$ | $CER_U - CER_C$ | $pv - boot$ | $pv - boot$ |
| Treasury | Short-term | 0.2891 | 1.5005 | -0.1912 | 0.3447 | 0.4812 | 1.1558 | 0.0460 | 0.0484 |
| | Long-term | 0.1286 | 0.5358 | 0.3632 | 0.6010 | -0.2346 | -0.0652 | 0.0479 | 0.0014 |
| TIPS | Short-term | 0.2188 | 0.3602 | 0.4114 | 0.9541 | -0.1926 | -0.5939 | 0.0480 | 0.0004 |
| | Long-term | 0.1399 | 0.3502 | 0.8102 | 0.8110 | -0.6703 | -0.4608 | 0.0404 | 0.0004 |

Panel B: Out-of-sample results

| | | Unconditional | | Conditional | | Differential | | $p - value$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | $SR$ | $CER$ | $SR$ | $CER$ | $SR_U - SR_C$ | $CER_U - CER_C$ | $pv - boot$ | $pv - boot$ |
| Treasury | Short-term | 0.2823 | 1.5886 | -0.2261 | 0.3204 | 0.5084 | 1.2682 | 0.0553 | 0.0548 |
| | Long-term | 0.1220 | 0.5540 | 0.3413 | 0.6456 | -0.2193 | -0.0916 | 0.0504 | 0.0040 |
| TIPS | Short-term | 0.2085 | 0.3351 | 0.3520 | 1.1332 | -0.1435 | -0.7981 | 0.0541 | 0.0005 |
| | Long-term | 0.1295 | 0.2894 | 0.7521 | 0.8846 | -0.7551 | -0.5952 | 0.0464 | 0.0006 |

This table reports the out-of-sample $CER$ returns for two different investor strategies: unconditional (bond returns are i.i.d) and conditional (bond returns are predictable) strategy. The $p - values$ of the difference between $SR$, and $CER$ from each strategy are obtained applying the Politis and Romano (1994) bootstrap procedure. The complete data set correspond to U.S. data from January 1, 2004 to December 30 2011.

Similarly, the difference between the in-sample $SR$ for the unconditional and conditional strategy shows the loss (given that I obtain negative values) from investing, based on the belief that bond returns are i.i.d. This means that the bond return predictability translates into improved in-sample asset allocation and performance. The comparison of in-sample certainty equivalent returns and their differences, confirms the conclusions from the analysis of Sharpe ratios. Finally, the difference between the Sharpe ratios and certainty equivalent returns of each strategy are statistically significant in all cases, as $pv - boot$ values indicate.

Next, I assess the magnitude of the potential gains that can actually be realized by an investor, using the out-of-sample performance of the strategies. From panel B of Table 6, we see that in all cases the $SR$ for the portfolios from the conditional strategy is much higher than for the unconditional strategy. I find the same results for $CER$. This means that a conditional strategy outperforms the unconditional strategy. This suggests also that conditional strategy might improve, not only in-sample but also out-of-sample performance. The significance of the $CER$ differential and the $SR$ differential, which is measure using the stationary bootstrap technique proposed by Politis and Romano (1994), implies that this result is statistically significant.

Finally, the difference between the in-sample and out-of-sample strategies allows me to gauge the severity of the estimation error. From the out-of-sample Sharpe ratio, reported in Panel B of Table 6, the unconditional strategy does not have a substantially lower Sharpe ratio and certainty equivalent returns out-of-sample than in-sample. This means that the effect of estimation error seems not to be so large. Consequently, it does not erodes the gains from optimal diversification given that differences turn out not to be economically important.

# 5 Conclusions

I consider the portfolio problem of a mean-variance and a power utility investor whose portfolio choices are between the asset of interest and a risk-free asset. The investor's problem is to choose optimal allocations to the risky asset as a function of predictor value: liquidity premium. The goal is to assess whether or not liquidity changes influence optimal portfolio allocations in the U.S. government bond market. While these issues have been well studied for stock-only portfolios, in general, less has been done to provide empirical evidence for the optimal portfolio choice of Foreign Official Institutions investing in U.S. Treasury securities, conditional upon observing a particular liquidity signal. This analysis is particularly important for central banks, specially in developing countries, given that collectively the have accumulate a large holdings of U.S. securities during the last fifteen years.

Overall, results show that optimal portfolios vary substantially with regards to predictor value. In particular, the effect of liquidity is a decreasing function of the investment horizon. Additionally, conditional allocations in risky assets decrease as liquidity conditions worsen. However, once the liquidity differential between U.S. nominal Treasury and TIPS bonds is sufficiently large, it leads to: (*i*) lower optimal portfolio allocations in TIPS; and (*ii*) higher optimal portfolio allocations on nominal bonds with respect to the risk-free bond. To summarize, this paper suggests that market liquidity signals could provide valuable guidance to central banks as one of the main FOIs investing in U.S. Treasury securities, and adds to the evidence found for stock portfolios by Ghysels and Pereira (2008), which suggests the existence of a dependence of the optimal portfolio choices on changes in liquidity.

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