A process calculus for spatially-explicit ecological models

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Background

- One trend of **theoretical ecology**: Individual-based modeling of ecosystems.
- **Individual-based modeling** is the opposite to population-based modeling
- Application area: **Metapopulations**
  - Local populations in spatially-separated habitat patches
  - Populations interact locally inside a patch
  - Individuals can disperse among patches
- Conservation ecology, species reintroduction
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  • Differential equations
  • Recurrence equations

• **Formal methods** individual-based modeling of ecological systems
  • Process calculi, P-systems, cellular automata
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Simulations carried out by ecologists often impose an order on the events that may take place within a model. Ordering can have implications on the simulation. Examples of temporal process ordering in ecological systems include:

- Concurrent ordering
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- Mortality before reproduction
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Our contributions

- Process Algebra with Locations for Population Systems (PALPS)
  - spatial calculus, locations, location attributes
  - location dependent behavior of individuals
  - Process ordering as a policy
  - semantics for a policy for actions
  - formal translation to model checker PRISM
  - simulation results
PALPS

• Basic entities
  • Individuals, Species, Locations, Channels and Attributes
Examples of expressions

- There is only one individual of species $s$ in myloc:
  $$s_{@myloc} = 1$$
- Temperature is less than 40 or Humidity is higher that 90 at location $\ell$:
  $$T_{@\ell} > 40 \lor H_{@\ell} > 90$$
- Total number of individuals at location $\ell$:
  $$s_{@myloc} + s'_{@myloc} < 10$$
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PALPS syntax (1)

• The individual level

\[ P ::= \begin{align*}
0 & \quad \text{inactive individual} \\
\sum_{i \in I} \eta_i . P_i & \quad \text{non-deterministic choice} \\
\sum_{i \in I} p_i : P_i & \quad \text{probabilistic choice} \\
\text{cond} \left( e_1 \triangleright P_1, \ldots, e_n \triangleright P_n \right) & \quad \text{conditional} \\
C & \quad \text{constant}
\end{align*} \]

• Actions

\[ \eta ::= a \mid \overline{a} \mid \text{go } \ell \mid \checkmark \quad \text{input,output,move,time} \]
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PALPS syntax (2)

• The species level

\[ R ::= !\text{rep}.P \]

• The system level

\[ S ::= 0 \quad \text{inactive system} \]
\[ P:⟨s, \ell⟩ \quad \text{located individuals} \]
\[ R:⟨s⟩ \quad \text{named species} \]
\[ S_1 | S_2 \quad \text{parallel composition} \]
\[ S\backslash L \quad \text{restriction} \]
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**PALPS semantics (1)**

- **Operational semantics** defined at the level of configurations \((E, S)\)
  - \(E\): an environment
  - \(S\): a population system
- The environment in needed to evaluate the expressions
- As an example, the initial environment for

\[
S \overset{\text{def}}{=} (P_0:⟨\ell, s, 2⟩|P_0:⟨\ell', s⟩|(\text{!rep}.P_0):⟨s⟩)\setminus \{\text{rep}\}.
\]

is

\[
E \overset{\text{def}}{=} \{(\ell, s, 2), (\ell', s, 1)\}
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PALPS semantics (2)

- Two transition relations
  - Probabilistic transition relation
    \[(E, S) \xrightarrow{w}^p (E', S')\]
  - Non-deterministic transition relation
    \[(E, S) \xrightarrow{\alpha}^n (E', S')\]
• The semantics is given at two levels
  • Individual level
  • System level
• Asynchronous communication
• All processes synchronize on the time passing actions
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Process ordering in PALPS (1)

- A policy $\sigma$ is a partial order on the set of PALPS non-probabilistic actions.
- A policy is set of tuples $(\alpha, \beta)$, where $\alpha, \beta$ are actions
- A policy models process ordering in ecological systems
Process ordering in **PALPS** (2)

- A prioritized transition relation

\[
(E, S) \xrightarrow{\alpha} (E', S') \quad \text{and} \quad (E, S) \xrightarrow{\beta}, (\alpha, \beta) \in \sigma
\]

\[
(E, S) \xrightarrow{\alpha, \beta} (E', S')
\]
Process ordering in PALPS (3)

- **Examples of policies in PALPS.** Let $\ell, \ell' \in \text{Loc},$
  
  - Concurrent ordering $\sigma = \{\}$
  - Reproduction before dispersal $\sigma = \{(\tau_{\text{rep},\ell,s}, \tau_{\text{go},\ell',s})\}$
  - Dispersal before reproduction $\sigma = \{(\tau_{\text{go},\ell',s}, \tau_{\text{rep},\ell,s})\}$
• Varroa-mite parasites live on an $n \times n$ lattice of honey-bee cells and cycle through the following.
  • **Death:** with probability $p$
  • **Dispersal:** randomly
  • **Reproduction:** produces an offspring of size $b$
PALPS example (2)
The individual level

\[ P_0 \overset{\text{def}}{=} p \cdot P_1 + (1 - p) : \sqrt{.0} \]

\[ P_1 \overset{\text{def}}{=} \sum_{\ell \in \text{Nb}(\text{myloc})} \frac{1}{4} : \text{go } \ell. \]

\[ \text{cond } (s@\text{myloc} = 1 \triangleright P_2; \text{true} \triangleright \sqrt{.0}) \]

\[ P_2 \overset{\text{def}}{=} \overline{\text{rep}}^b \cdot \sqrt{.0} \]

where \( \overline{\text{rep}}^b \overset{\text{def}}{=} \overline{\text{rep}} \ldots \overline{\text{rep}} \)

\( b \) times
PALPS example (4)

- The species level
  \[ R \overset{\text{def}}{=} !\text{rep}.P_0 \]

- The system level
  \[ \text{System} \overset{\text{def}}{=} (P_0:⟨\ell, s, 2⟩|P_0:⟨\ell', s⟩|(!\text{rep}.P_0):⟨s⟩) \setminus \{\text{rep}\}. \]
We use the policy \textbf{dispersal before reproduction} 
\[ \{ (T_{\text{rep}, \ell, s}, T_{\text{go}, \ell', s}) | \ell, \ell' \in \text{Loc} \} \] for this example.
• Semantics of the policy **dispersal before reproduction** \( \{(\tau_{\text{rep},\ell,s}, \tau_{\text{go},\ell',s})\} \) for the example.
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Encoding of PALPS into PRISM (1)

- PRISM is a probabilistic model checker\(^1\)
- To translate PALPS into the PRISM language
  - each process is a module
  - the execution flow is captured by a local variable
  - all processes synchronize on the $\sqrt{}$ action

\(^1\)www.prismmodelchecker.org/
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Encoding of **PALPS into PRISM (2)**

- To translate **PALPS** into the **PRISM** language
  - we map binary communication into multi-way communication
  - replication is bounded
  - we define a global variable for each action to ensure the semantics of the policy
Encoding of **PALPS into PRISM (3)**

**Correctness**

For any configuration \((E, Sys)\) and policy \(\sigma\), where \(E\) is compatible with \(Sys\), whenever \((E, Sys) \xrightarrow{\alpha} (E', Sys')\) then 

\[
\llbracket (E, Sys) \rrbracket \xrightarrow{m} \llbracket (E', Sys') \rrbracket
\]

where \(1 \leq m \leq 3\).

- A similar result holds in the opposite direction.
Model checking of PALPS using PRISM (1)

- Verification of probabilistic temporal PCTL properties
  - Probability of extinction of the population in the next 10 years is less than a certain threshold $p_e$
  - Within the next 20 years with some high probability, members of the population $s$ will outnumber the members of population $s'$
  - Compare the average number of individuals of species $s$ at time unit $t$ to a constant
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Model checking of **PALPS using PRISM (2)**

- **Semantics of model checking**
  - **Defined over Markov Decision Processes:** Computes minimum and maximum probabilities
  - **Approximation defined over Discrete-Time Markov Chains:** Computes reward-based properties, steady state and reachability
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Simulation of **PALPS using PRISM**

- Explore random paths of execution
- Search for deadlocks using **PRISM** simulation
- Perform **model-checking by simulation**
## Results for the example (1)

<table>
<thead>
<tr>
<th>Case study size</th>
<th>Number of States</th>
<th>Construction time (sec.)</th>
<th>RAM (GB)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No policy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 PALPS individuals</td>
<td>130397</td>
<td>8</td>
<td>0.5</td>
</tr>
<tr>
<td>4 PALPS individuals</td>
<td>1830736</td>
<td>101</td>
<td>1.9</td>
</tr>
<tr>
<td><strong>Policy $\sigma$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 PALPS individuals</td>
<td>27977</td>
<td>3</td>
<td>0.3</td>
</tr>
<tr>
<td>4 PALPS individuals</td>
<td>148397</td>
<td>10</td>
<td>0.7</td>
</tr>
<tr>
<td><strong>Extended policy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 PALPS individuals</td>
<td>20201</td>
<td>3</td>
<td>0.3</td>
</tr>
<tr>
<td>4 PALPS individuals</td>
<td>128938</td>
<td>9</td>
<td>0.6</td>
</tr>
</tbody>
</table>

**Table:** Performance of building probabilistic models in PRISM with and without policies.
Results for the example (2)

- Applying a policy $\sigma = \{ (\tau_{rep, \ell, s}, \tau_{go, \ell', s}) | \ell, \ell' \in \text{Loc} \}$ reduced the size of the state space by a factor of 10.
- Applying a policy for the execution of actions among individuals reduced the state space by about 20% more.
Results for the example (2)

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• Applying a policy for the execution of actions among individuals reduced the state space by about 20% more
Results for the example (3)

- Results obtained using statistical model checking
  - Using simulation to verify a $\text{PCTL}$ property
Results for the example (4)

- Expected population size vs simulation time for different initial sizes of the population, with offspring size $b = 2$.

- The total number of individuals after a “long time” is independent of the initial number of individuals.
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Results for the example (5)

• Expected population size vs simulation time for different offspring sizes. Probability to die $p = 0.1$ and initial population $i = 1$.

• For $b > 2$, the total number of individuals is periodic until extinction.
Results for the example (5)

- Expected population size vs simulation time for different offspring sizes. Probability to die $p = 0.1$ and initial population $i = 1$.

![Graph showing average total number of individuals per time unit](image)

- For $b > 2$, the total number of individuals is periodic until extinction.
Reducing the state space (1)

- We reduced the state space of PALPS models with policies, but
  - for some applications, it is still too big

- Proposed solution
  - Synchronous communication [3]
  - Mean-field semantics
Reducing the state space (1)

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  • Synchronous communication [3]
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Reducing the state space (2)

- Complete state space
Reducing the state space (3)

- State-space reduced with a policy
Reducing the state space (4)

- State-space reduced with synchronous communication
Conclusions

- **PALPS**
  - Discrete space, discrete time, probabilistic behavior
  - Location attributes and location-dependent behavior
  - **Policies** that
    - Reduce the state space
    - Allow to model different process orderings
  - Semantics for **PALPS** with synchronous communication
  - Support for simulation and analysis of models through PRISM translation
Future work

- Mean-field semantics à la WSCCS
• **T H A N K Y O U**

Do you have any question?
References

