Reasoning about Functional Programs by Combining Interactive and Automatic Proofs

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(joint work with Ana Bove\(^2\) and Peter Dybjer\(^2\))

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Our Goal

To build a computer-assisted framework for reasoning about programs written in Haskell-like pure and lazy functional languages.
Some Paradigms of Programming

**Imperative:** Describe computation in terms of state-transforming operations such as assignment. Programming is done with statements.

**Logic:** Predicate calculus as a programming language. Programming is done with sentences.

**Functional:** Describe computation in terms of (mathematical) functions. Programming is done with expressions.
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**Examples**

<table>
<thead>
<tr>
<th>Imperative</th>
<th>Logic</th>
<th>Functional</th>
</tr>
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<tbody>
<tr>
<td>C</td>
<td>CLP(R)</td>
<td>Standard ML</td>
</tr>
<tr>
<td>C++</td>
<td>Prolog</td>
<td>Erlang</td>
</tr>
<tr>
<td>Java</td>
<td></td>
<td>Clean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pure</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
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<td>Idris</td>
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Side effects

“A side effect introduces a dependency between the global state of the system and the behaviour of a function ... Side effects are essentially invisible inputs to, or outputs from, functions.”

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In Haskell all the functions are pure functions, i.e. they “take all their input as explicit arguments, and produce all their output as explicit results.”

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Haskell: A Pure Functional Programming Language

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Referential transparency

Equals can be replaced by equals.

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“The first program to write is the same for all languages: Print the words *hello, world.*” (1978, §1.1)
Example

The following C program prints "hello, world" twice.

```c
#include <stdio.h>

int main (void) {
    printf ("hello, world");
    printf ("hello, world");

    return 0;
}
```
Example
The following C program prints "hello, world" once.

```c
#include <stdio.h>

int main (void)
{
    int x;

    x = printf ("hello, world");
    x; x;

    return 0;
}
```
Example (Lists)

Haskell has built-in syntax for lists, where a list is either:

- the empty list, written [], or
- a first element \(x\) and a list \(xs\), written \(\text{length } (x : xs)\).
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Example (Pattern matching on lists)

\[
\text{length} :: [\text{Int}] \rightarrow \text{Int} \\
\text{length} \ [ ] = 0 \\
\text{length} \ (x : xs) = 1 + \text{length} \ xs
\]
Example (Parametric polymorphism)

\[
\text{length :: } [a] \rightarrow \text{Int} \\
\text{length } [] = 0 \\
\text{length } (x : xs) = 1 + \text{length } xs
\]
Haskell: A Pure Functional Programming Language

Lazy evaluation
Nothing is evaluated until necessary.

Example: 

```haskell
take ∷ [Int] → [a] → [a]
squares ∷ [Int]
squares = [x^2 | x ← [1..]]
```

Which is the value of `take 5 squares`? 

\[ 1, 4, 9, 16, 25 \]
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Question

What if we have written a Haskell-like program and we want to verify it?

Remark:
Most of the proof assistants lack a direct treatment for general recursive functions.

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A. Sicard-Ramírez
Question

What if we have written a Haskell-like program and we want to verify it?

How to deal with the possible use of general recursion?

(non-structural recursive, nested recursive, and higher-order recursive functions, and guarded and unguarded co-recursive functions)

Remark: Most of the proof assistants lack a direct treatment for general recursive functions.³

Programming Logics

Programming logic

A logic in which programs and specifications can be expressed and in which it can be proved or disproved that a certain program meets a certain specification.
Proof Assistant

An interactive computer system which helps with the development of formal proofs.

Examples (incomplete list)

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<td>2.4.2 (Aug. 2014)</td>
<td>Haskell</td>
<td>Type theory</td>
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Automatising First-Order Logic Proofs

Automatic theorem provers for first-order logic (ATPs)

- **TPTP**: a language understood by many off-the-shelf ATPs
- The **TPTP** world: [http://www.cs.miami.edu/~tptp/](http://www.cs.miami.edu/~tptp/)
- The CADE ATP System Competition
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- **Programs**: Type-free extended versions of Plotkin’s PCF language
- **Specification language**: First-order logic and predicates representing the property of being a finite or a potentially infinite value
- **Inference rules**: Conversion and discrimination rules for the term language, introduction and elimination for the (co)-inductive predicates
Our Main Contributions

2. What proof assistant should we use?

We formalise our programming logic and our examples of verification of functional programs in the Agda proof assistant: we use Agda as a logical framework (meta-logical system for formalising other logics) and we use Agda’s proof engine:

i) support for inductively defined types including inductive families, and function definitions using pattern matching on such types,

ii) normalisation during type-checking,

iii) commands for refining proof terms,

iv) coverage checker and

v) termination checker.
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- we provide a translation of our Agda representation of first-order formulae into TPTP so we can use them when proving the properties of our programs,
- we extended Agda with an ATP-pragma, which instructs Agda to interact with the ATPs, and
- we wrote the Apia program, a Haskell program which uses Agda as a Haskell library, performs the above translation and calls the ATPs.
Related Publications


The programs and examples described are available as Git repositories at GitHub:

- The extended version of Agda: https://github.com/asr/eagda.
- The Agda implementation of our programming logics, some first-order theories and examples of verification of functional programs: https://github.com/asr/fotc.
Thanks!