First-Order Proof Reconstruction
(Research Proposal – 2016)

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Abstract

In a previous research, we proposed a first-order theory for reasoning about functional programs by combining interactive proofs performed in the Agda proof assistant and automatic proofs performed by off-the-shelf first-order automatic theorem provers (ATPs). Our approach can be used with other first-order theories too. We have used it with other first-order theories such as Group Theory and Peano Arithmetic, and we had encouraging results. In our approach, we use the ATPs as oracles via a Haskell program called Apia, that is, we trust the ATPs when they tell us that a proof exists. In consequence, the consistency of our approach relies on the correct implementation of both the Apia program and the ATPs. We propose strengthen the consistency of our approach by reconstructing in Agda the first-order proofs automatically produced.
Project Information

Team work

- Andrés Sicard-Ramírez (main researcher)
- Juan Fernando Ospina-Giraldo (co-researcher)
- Research assistant (student of the Master in Applied Mathematics)

Subject

Formalisation of proofs, verification of functional programs, type theory, proof assistants and automatic theorem provers.
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Formalisation of proofs, verification of functional programs, type theory, proof assistants and automatic theorem provers.
Context of Our Research Problem

- Verification of programs


  - Verification of compilers


  - Programming logic (a logic in which programs and specifications can be expressed and in which it can be proved or disproved that a certain program meets a certain specification).
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  Example: CompCert Project (2008 - current)
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  - Dependent types (a dependent type is a type that depend on a value).

- Π-types
  \[ \Pi x : A. B(x) \] is the type of terms \( f \) such that, for every \( a : A \) then \( f \ a : B(a) \).

- Σ-types
  \[ \Sigma x : A. B(x) \] is the type of pairs \((m, n)\) such that \( m : A \) and \( n : B(m) \).
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  - ATPs for first-order logic
    - The TPTP world ([http://www.cs.miami.edu/~tptp/](http://www.cs.miami.edu/~tptp/)).
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- Apia
  - A Haskell program which:
    (i) provides a translation of our Agda representation of first-order formulae into TPTP languages (FOF, TFF0) and
    (ii) calls the ATPs.
Research Problem

Problem

In our approach to the verification of functional programs [Bove, Dybjer, and Sicard-Ramírez 2009, 2012; Sicard-Ramírez 2014], we use the ATPs as oracles via the Apia program, that is, we trust the ATPs when they tell us that a proof exists.

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Goal

Reconstruct first-order proofs produced by one ATP using Agda as an logical framework.
State of Art

- We do not know of any existing first-order proof reconstruction in the Agda proof assistant.
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- Sledgehammer provides a full integration of automatic theorem provers including ATPs for first-order logic and SMT solvers [Blanchette, Böhme, and Paulson 2013] with Isabelle/HOL.
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- Foster and Struth [2011] integrate Waldmeister into Agda. This integration uses a proof reconstruction step. The approach is restricted to pure equational logic—FOL with equality but no other predicate symbols and no functions symbols [Appel 1959].
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- **SMTCoq** [Armand et al. 2011] is a tool for the **Coq** proof assistant which provides a certified checker for proof witnesses coming from the SMT solver **veriT** and adds a new tactic named **verit**, that calls **veriT** on any **Coq** goal.
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References


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Thanks!