Inspire Create Transform
Wavefield separation cross-correlation imaging condition based on continuous wavelet transform

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Laguerre-Gauss transform in post-processing imaging

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References
Reverse time migration (RTM)

\[ I_t = \sum_t P_s P_r \]

**Acquisition**

**Shots 1, ..., n**

**Imaging condition**

**Final migrated section**

**Forward modeling**

**Pr**

**Backward modeling**

**Ps**

**Migrated shots**

\[ I = \sum_{i=1}^{n} I_t \]
Reverse time migration (RTM)

Acoustic wave equation

\[
\frac{1}{c(x)^2} \frac{\partial^2 u(x, t)}{\partial t^2} - \nabla^2 u(x, t) = s(x, t)
\]

1. Forward propagation of the source wavefield.
2. Backward propagation of the receivers wavefield.
3. Imaging condition.
Reverse time migration (RTM)

Velocity model

Data recorded
Source wavefield video  Receiver wavefield video
\[ t = 0.20 \text{ s} \]

\[ t = 0.36 \text{ s} \]
Zero-lag cross-correlation imaging condition (ZL-CC-IC)

\[ I_{cc}(x, z) = \sum_{j=1}^{s_{max}} \sum_{i=1}^{t_{max}} S(x, z; t_i; s_j) R(x, z; t_i; s_j) \]  

\( S \): Source wavefield  
\( R \): Receiver wavefield  
\( z \): Depth  
\( x \): Distance  
\( t \): Time  
\( t_{max} \): Maximum time  
\( s_{max} \): Maximum number of sources
Cross-correlation image
Cross-correlation image
Some wave paths of the wavefield [22]
Methods to eliminate the artifacts

- Wavefield propagation approaches ([25, 3, 12]).
- Imaging condition approaches ([38, 20, 17, 22, 43, 31, 35]).
- Post-imaging condition approaches ([45, 16]).
Laguerre-Gauss transform

The Laguerre-Gauss transform of $I(x, y)$ is given by ([41, 15])

$$\tilde{I}(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} LG(f_x, f_y) I(f_x, f_y) e^{2\pi i (f_x x + f_y y)} df_x df_y$$

Where

$$LG(x, y) = (i\pi^2\omega^4)(x + iy) e^{-\pi^2\omega^2(x^2 + y^2)}$$
Cross-correlation image

Laguerre-Gauss image
Wavefield decomposition

Taking into account (1)

\[ I_{cc}(x, z) = \sum_{j=1}^{s_{\text{max}}} \sum_{i=1}^{t_{\text{max}}} S(x, z; t_i; s_j) R(x, z; t_i; s_j) \]

\( S(x, z; t_i; s_j) \) and \( R(x, z; t_i; s_j) \) can be partitioned mathematically as

\[ S(x, z; t_i; s_j) = S_d(x, z; t_i; s_j) + S_u(x, z; t_i; s_j) \]

\[ R(x, z; t_i; s_j) = R_d(x, z; t_i; s_j) + R_u(x, z; t_i; s_j) \]
Wavefield decomposition

Then, (1) can be expressed as follows

\[ I_{cc}(x, z) = \sum_{j=1}^{s_{\text{max}}} \sum_{i=1}^{t_{\text{max}}} (S_d(x, z; t_i; s_j)R_u(x, z; t_i; s_j) + S_u(x, z; t_i; s_j)R_d(x, z; t_i; s_j) + S_d(x, z; t_i; s_j)R_d(x, z; t_i; s_j) + S_u(x, z; t_i; s_j)R_u(x, z; t_i; s_j)) \]

Then

\[ I_{cc}(x, z) = I_{cc}^{du}(x, z) + I_{cc}^{ud}(x, z) + I_{cc}^{dd}(x, z) + I_{cc}^{uu}(x, z) \]  (2)
Wavefield decomposition

From (2)

\[ I_{cc}(x, z) = I_{cc}^{du}(x, z) = \sum_{j=1}^{s_{max}} \sum_{i=1}^{t_{max}} S_d(x, z; t_i; s_j) R_u(x, z; t_i; s_j) \]  (3)

\( S_d(x, z; t_i; s_j) \): Downgoing source wavefield.

\( R_u(x, z; t_i; s_j) \): Upgoing receiver wavefield.

Eq. (3) is exactly what one will get in a one way wave equation migration.
Continuous wavelet transform

A wavelet is a function $\psi \in L^2(\mathbb{R})$ with finite energy ([27]), that is,

$$C_\psi = \int_0^\infty \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty$$

$\hat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$ given by

$$\hat{\psi}(\omega) = \int_{-\infty}^{\infty} \psi(t)e^{-i2\pi\omega t} dt$$

$C_\psi$ is called the admissibility condition.
Continuous wavelet transform

It is normalized $\|\psi\| = 1$ and satisfies the condition that is rapidly decreasing

$$\int_{-\infty}^{\infty} (1 + |t|)|\psi(t)|dt < \infty$$

with zero average and centered in the neighborhood of $t = 0$

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$
Continuous wavelet transform

Family of wavelets

\[ \psi_{s,u}(t) = \frac{1}{\sqrt{s}} \psi \left( \frac{t - u}{s} \right), \quad s, u \in \mathbb{R}, \quad a \neq 0 \]

\( s \): Scaling parameter
\( u \): Translation parameter
\( \psi \): Mother wavelet

If \( \psi \in L^2(\mathbb{R}) \), then \( \psi_{s,u}(t) \in L^2(\mathbb{R}) \) for all \( s, u \) and \( ||\psi_{s,u}|| = 1 \).
Continuous wavelet transform

The integral transformation $W_f$ defined on $L^2(\mathbb{R})$ by

$$W_f(u, s) = \langle f(t), \psi_{s,u}(t) \rangle = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t) \psi^* \left( \frac{t-u}{s} \right) dt$$

is called a continuous wavelet transform of $f(t)$.

Gaussian wavelet

$$\psi_n(t) = c_n \frac{d^n}{dt^n} \left( e^{-\frac{t^2}{2}} \right), \quad \hat{\psi}_n(\omega) = c_n (i\omega)^n e^{-\frac{\omega^2}{4}}$$
Continuous wavelet transform

The continuous wavelet transform can be expressed as a convolution product

\[ W_f(u, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t) \psi^* \left( \frac{t-u}{s} \right) dt = f(t) \ast \psi_s(u) \]

with

\[ \psi_s(t) = \frac{1}{\sqrt{s}} \psi^* \left( \frac{-t}{s} \right) \]

and the Fourier transform of \( \psi_s(t) \) is

\[ \hat{\psi}_s(\omega) = \sqrt{s} \hat{\psi}^*(s\omega) \]
Source wavefield analysis

Algorithm for time-scale analysis of source wavefield

- From the source wavefield $S(x, z, t)$, select for each $x$ the wavefield $S_x(z, t)$. 
Source wavefield analysis

$S(x, z, t)$ wavefield

$S_x(z, t)$ at $x = 0.90$ km
a) $x = 0.375 \text{ km}$

b) $x = 0.90 \text{ km}$

c) $x = 1.125 \text{ km}$

d) $x = 2.25 \text{ km}$
Source wavefield analysis

Algorithm for time-scale analysis of source wavefield

- From the source wavefield $S(x, z, t)$, select for each $x$ the wavefield $S_x(z, t)$.
- Apply 1D CWT on $S_x(z, t)$ along $t$ axis for each $z$ ($S_{x,z}(t)$).
$S_{x=1.125,z}(t)$ at $z = 0 \text{ km}$

$S_{x=1.125,z}(t)$ at $z = 0.45 \text{ km}$

Coefficients of CWT
Source wavefield analysis

Algorithm for time-scale analysis of source wavefield

- From the source wavefield $S(x, z, t)$, select for each $x$ the wavefield $S_x(z, t)$.
- Apply 1D CWT on $S_x(z, t)$ along $t$ axis for each $z$ ($S_{x,z}(t)$).
- Select the minimum value of the all coefficients and locate it in $S_{x,z}(t)$ and saved in a new wavefield $S_{x,z}^{\text{new}}(t)$. Two more points were taken before and after this point to improve the accuracy.
$S_x(z, t)$ at $x = 1.125 \text{ km}$

$S^{new}_x(t)$ at $x = 1.125 \text{ km}$

$S_x(z, t)$ at $x = 2.25 \text{ km}$

$S^{new}_x(z, t)$ at $x = 2.25 \text{ km}$
Source wavefield analysis

\[ S(x, z, t) \] wavefield video  Separated \[ S(x, z, t) \] video
Receiver wavefield analysis

Algorithm for time-scale analysis of receiver wavefield

- From the receiver wavefield $R(x, z, t)$, select for each $x$ the wavefield $R_x(z, t)$. 
Receiver wavefield analysis

$R(x, z, t)$ wavefield

$R_x(z, t)$ at $x = 1.125$ km
a) $x = 0.675$ km

b) $x = 1.125$ km

c) $x = 1.50$ km

d) $x = 2.25$ km
Receiver wavefield analysis

Algorithm for time-scale analysis of receiver wavefield

- From the receiver wavefield \( R(x, z, t) \), select for each \( x \) the wavefield \( R_x(z, t) \).
- Apply 1D CWT on \( R_x(z, t) \) along \( z \) axis for each \( t \) (\( R_{x,t}(z) \)).
Receiver wavefield analysis

\[ R_{x=1.125,t}(z) \text{ at } t = 0.48 \text{ s} \]

\[ R_{x=1.125,t}(z) \text{ at } t = 0.66 \text{ s} \]
Receiver wavefield analysis

Algorithm for time-scale analysis of receiver wavefield

- From the receiver wavefield $R(x, z, t)$, select for each $x$ the wavefield $R_x(z, t)$.
- Apply 1D CWT on $R_x(z, t)$ along $z$ axis for each $t$ ($R_{x,t}(z)$).
- Select the maximum absolute value of coefficients that corresponds to a coefficient with negative value and locate it in $R_{x,t}(z)$ and saved in a new wavefield $R_{x,t}^{\text{new}}(z)$. Two more points were taken before and after this point to improve the accuracy.
$R_x(z, t)$ at $x = 1.125$ km

$R_{x,t}^{\text{new}}(z)$ at $x = 1.125$ km

$R_x(z, t)$ at $x = 2.25$ km

$R_{x,t}^{\text{new}}(z)$ at $x = 2.25$ km
Receiver wavefield analysis

Receiver wavefield video   Separated $R(x, z, t)$ video
Separated source and receiver wavefields

Separated $S(x, z, t)$ video  Separated $R(x, z, t)$ video
Cross-correlation image

Conventional cross-correlation image

Wavefield separation cross-correlation image
Cross-correlation image

Conventional cross-correlation image

Wavefield separation cross-correlation image
Other synthetic models

Three-layer model

Small salt model
Three-layer model

\[ S(x, z, t) \] video  \quad \text{Separated} \ S(x, z, t) \text{ video}
Three-layer model

\[ R(x, z, t) \text{ video} \quad \text{Separated} \quad R(x, z, t) \text{ video} \]
Three-layer model

Cross-correlation image

Conventional ZL-CC-IC  ZL-CC-IC with separated wavefield
Three-layer model

Receiver wavefield analysis

\[ R(x, z, t) \]

\[ R(x, z, t) \text{ at } x = 0.9 \text{ km} \]
Three-layer model

Receiver wavefield analysis

\[ R(x, z, t) \text{ at } x = 0.9 \text{ km} \]

Coefficients CWT \( R(x = 0.9, z, t) \) at \( t = 0.54 \text{ s} \)
Wavelet transform modulus maxima (WTMM)

WTMM corresponds to the entire set of local maximum points of the absolute value of wavelet transform.

\[ WTMM = \left\{ (u_0, s_0) \in (\mathbb{R}, \mathbb{R}^+) \mid \frac{\partial |W_f(u, s)|}{\partial u} \bigg|_{u=u_0, s=s_0} = 0 \right\} \]

The set of points of the WTMM concatenated through scales are known as maximum lines.
Wavelet transform modulus maxima (WTMM)

Local maximum points

Maximum lines chaining
Three-layer model

Cross-correlation image

Conventional ZL-CC-IC

ZL-CC-IC with separated source wavefield
Small salt model

\[ S(x, z, t) \text{ video} \quad \text{Separated } S(x, z, t) \text{ video} \]
Small salt model

Cross-correlation image

Conventional ZL-CC-IC  ZL-CC-IC with separated source wavefield
Preliminary results

Preliminary results


The use of Laguerre-Gauss transform in 2D reverse time migration imaging

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Abstract

Zero-lag cross-correlation imaging condition (C2-CC) is widely used in seismic data processing for the recovery of structural images of the subsurface. This condition presents spatial low frequency noise as called "sidelobes". The use of Laguerre-Gauss transform in 2D reverse time migration (RTM) imaging can provide better suppression of these complex low frequency noise features. In this paper, we present a cross-correlation approach using the Laguerre-Gauss transform (L-GT) for imaging. We performed synthetic 2D land modeling using zero-lag cross-correlation and the L-GT model to recover the structural images. The recovered images were compared to the images obtained using the conventional cross-correlation method (C2-CC). The results show that the Laguerre-Gauss transform improves the quality of the recovered images by providing better structural recovery and more clear structural details.

Introduction

The imaging condition in seismic data migration has been theoretically obtained using the zero-lag cross-correlation (C2-CC) by examining the spatial variations of the products of seismic amplitudes between the source and the receiver. This condition can have an important impact on the results obtained from the imaging condition. (Claerbout, 1975; 1985) This imaging condition is known as the "sidelobe" effect. The "sidelobe" effect produces the spatial low frequency noise features in the seismic images. (Lu et al., 2013) In this study, we use the Laguerre-Gauss transform (L-GT) to provide better suppression of these low frequency features in the seismic images.

However, the image is contaminated with low frequency background noise. The Laguerre-Gauss transform (L-GT) can be used to suppress this background noise and improve the quality of the structural images. (Chocpithay and Sathaborn, 2015) This imaging condition (C2-CC) can be used to suppress the low frequency noise background noise and improve the quality of the structural images. (Chocpithay and Sathaborn, 2015)

For small impedance contrasts, the cross-correlation is a good approximation for the imaging condition. However, for large impedance contrasts the low frequency artificial becomes more prominent as it becomes the dominant feature in the imaging condition. (Chocpithay and Sathaborn, 2015) In this study, we compare and analyze the results obtained using the conventional cross-correlation method and the L-GT method for imaging.

In this paper, we show some special features of the L-GT method in the post-processing of the structural images. The results obtained using the L-GT method improve the quality of the structural images. (Claerbout, 1975; 1985) The L-GT method provides better suppression of the low frequency noise background noise and improves the quality of the structural images. (Chocpithay and Sathaborn, 2015)

First, we compare and analyze the Fourier spectra obtained using the conventional cross-correlation method and the L-GT method. This comparison shows that the L-GT method improves the quality of the structural images.

The results obtained using the L-GT method show that the L-GT method provides better suppression of the low frequency noise background noise and improves the quality of the structural images. (Claerbout, 1975; 1985) The L-GT method provides better suppression of the low frequency noise background noise and improves the quality of the structural images. (Chocpithay and Sathaborn, 2015)
Preliminary results


LAGUERRE-GAUSS FILTERS IN REVERSE TIME MIGRATION IMAGE RECONSTRUCTION
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ABSTRACT
Reverse time migration (RTM) solves the acoustic or elastic wave equation by means of the extrapolation from source and receiver wavefields in time. A migrated image is obtained by applying a criteria known as imaging condition. The cross-correlation between source and receiver wavefields is the commonly used imaging condition. However, this imaging condition produces spatial low-frequency noise, called artifacts, due to the unwanted correlation of the diving, head and backscattered waves. Several techniques have been proposed to reduce the artifacts occurrence. Derivative operators as Laplacian are the most frequently used. In this work, we propose a technique based on a spatial phase filter ranging from 0 to 2π, and a temporal amplitude bandpass filter, known as Laguerre-Gauss transform. Through numerical experiments we present the application of this particular filter on three synthetic data sets. In addition, we present a comparative spectral study of images obtained by the zero-lag cross-correlation imaging condition, the Laplacian filtering and the Laguerre-Gauss filtering, showing their frequency features. We also present evidences not only with simulated noisy velocity fields but also by comparison with the model velocity field gradients that this method improves the RTM images by reducing the artifacts and notably enhance the reflective events.

Keywords: Laguerre-Gauss transform, zero-lag cross-correlation, seismic migration, imaging condition.

FILTROS DE LAGUERRE-GAUSS EM IMPRESSÃO DE IMAGEM DE MIGRAÇÃO DE TEMPO INVERSO

RESUMO
A migração reversa no tempo (RTM) resolve a equação de onda acústica ou elástica por meio da extrapolação a partir do campo de onda da fonte e do receptor no tempo. Uma imagem migrada é obtida aplicando um critério conhecido como condição de imagem. A correlação cruzada entre campos de onda de fonte e receptor é a condição de imagem comumente usada. No entanto, esta condição de imagem produz ruído espacial de baixa frequência, chamado artefatos, devido à correlação indesejada das ondas de reflexão, cablagem e refletidos. Várias técnicas têm sido propostas para reduzir a ocorrência de artefatos. Operadores derivados como Laplaciano são os mais utilizados. Neste trabalho, propomos uma técnica baseada em um filtro de fase aparente que varia de 0 a 2π, e um filtro passa-banda de amplitude toroidal, conhecido como transformada de Laguerre-Gauss. Através de experimentos numéricos, apresentamos a aplicação deste filtro particular em três
Preliminary results


Abstract: In this paper we introduce the singularity spectrum algorithm of a seismogram and analyze the features time-scale of the traces and the superposition of the scalogram of the complete set of traces, extracting there main features in time-scale domain that provide clues for the possible localization of artifacts that appears by Zero Lag Cross-Correlation imaging condition (ZLCC) of the operator Reverse Time Migration (RTM). We also tested the post-processing of ZLCC by Laplacian Filtering and Laguerre-Gauss Filtering (Paniagua and Sierra, 2016) and compare their localization and artifact removal capabilities in terms of the features found by Singularity Analysis.

Keywords: Wavelet Transform, Modulus Maxima, Zero-Lag Cross-Correlation, Laguerre-Gauss Filter, and seismic migration.

Introduction: Reverse time migration (RTM) is a very well-known technique for the retrieval of images of the subsurface from the solution of the acoustic wave equation for wavefield propagation through a
Future work

- Analyze and study the extraction of information about the upgoing and downgoing components of source and receiver wavefields obtained through the CWT and WTMM.
- Analyze the features of the coefficients in CWT and WTMM of the signals in order to select and extract the information properly.
- Improve the algorithm to extract the relevant information about source and receiver wavefields in multi-layer synthetic models.
- Apply the proposed method in migrations with multiple shots.
Future work

- Extend the implementation of the algorithm to other complex synthetic models.
- Realize a singularity analysis of wavefields in order to find the relationship between the local maximum points and lines chaining, obtained by CWT and WTMM, with the Hölder exponent. (Our hypothesis is that there is a relationship between the local maximum points and the Hölder exponent)
References


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