Seminar of the PhD in Mathematical Engineering
Universidad EAFIT

Background Error Estimation In Sequential Data Assimilation

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Motivation I

- Weather forecasts and warnings are the most important services provided by the meteorological profession.
- Forecasts are used by
  - Government and industry to protect life and property.
  - To improve the efficiency of operations.
  - Individuals to plan a wide range of daily activities.
- Weather forecasting today is a highly developed skill:
  - It is grounded in scientific principles and methods.
  - Makes use of advanced technological tools.
- **How do we forecast the state of (highly non-linear) dynamical system?**
  - An imperfect numerical forecast.
  - Observations of the actual state.
  - Observation operator.
Components in DA [BS12] I

- We want to estimate \( \mathbf{x}^* \in \mathbb{R}^{n \times 1} \). \( n \sim \mathcal{O}(10^8) \).

- Imperfect numerical model:

\[
\mathbf{x}_{\text{next}} = \mathcal{M}_{t_{\text{current}} \to t_{\text{next}}} (\mathbf{x}_{\text{current}}),
\]

where \( \mathbf{x} \in \mathbb{R}^{n \times 1} \).

- Noisy observations:

\[
\mathbf{y} = \mathcal{H} (\mathbf{x}) + \mathbf{\epsilon} \in \mathbb{R}^{m \times 1},
\]

where \( \mathcal{H} : \mathbb{R}^n \to \mathbb{R}^m \) and \( \mathbf{\epsilon} \sim \mathcal{N}(\mathbf{0}_m, \mathbf{R}). m \sim \mathcal{O}(10^6) \).

- Prior estimate \( \mathbf{x}^b \in \mathbb{R}^{n \times 1} \) with errors following \( \mathcal{N}(0, \mathbf{B}) \).
Components in DA [BS12] II

(a) $x^*$

(b) $H$

(c) $y = H \cdot x^* + \epsilon$

(d) $x^b$

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By Bayes’ Theorem we know that:

\[ P(x|y) \propto P(x) \cdot L(x|y) \]

where

\[ P(x) \propto \exp \left( -\frac{1}{2} \cdot \| x - x^b \|^2_B \right) \]

\[ L(x|y) \propto \exp \left( -\frac{1}{2} \cdot \| y - H \cdot x \|^2_R \right) \]

and therefore,

\[ x^a = \arg \max_x P(x|y), \]


Components in DA [BS12] IV

- It can be easily shown that:

\[
 x^a = x^b + A \cdot H^T \cdot R^{-1} \cdot d = A \cdot \left[ B^{-1} \cdot x^b + H^T \cdot R^{-1} \cdot y \right] \\
= x^b + B \cdot H^T \cdot \left[ R + H \cdot B \cdot H^T \right]^{-1} \cdot d
\]

where \( A = \left[ B^{-1} + H^T \cdot R^{-1} \cdot H \right]^{-1} \in \mathbb{R}^{n\times n} \), and \( d = y - H \cdot x^b \in \mathbb{R}^{m\times 1} \).

- Posterior distribution:

\[
x \sim \mathcal{N}(x^a, A).
\]
Sequential Data Assimilation Problem

Figure: Sequential Data Assimilation process.

At assimilation steps, we do need to estimate $x^b$ and $B$ (moments of the prior error distribution).
We can make use of an ensemble of model realizations:

\[ \mathbf{X}^b = \left[ \mathbf{x}^{b[1]}, \mathbf{x}^{b[2]}, \ldots, \mathbf{x}^{b[N]} \right] \in \mathbb{R}^{n \times N} \]

Empirical moments of the ensemble:

\[ \mathbf{x}^b \approx \bar{\mathbf{x}}^b = \frac{1}{N} \cdot \mathbf{X}^b \cdot \mathbf{1}_N \in \mathbb{R}^{n \times n}, \]

\[ \mathbf{B} \approx \mathbf{P}^b = \frac{1}{N - 1} \cdot \mathbf{\delta X} \cdot \mathbf{\delta X}^T, \]

and

\[ \mathbf{\delta X} = \mathbf{X}^b - \bar{\mathbf{x}}^b \cdot \mathbf{1}_N^T \in \mathbb{R}^{n \times N}. \]
The Lorenz 96 Model - Toy Model I

The Lorenz 96 model:

\[
\frac{dx_j}{dt} = \begin{cases} 
(x_2 - x_{n-1}) \cdot x_n - x_1 + F & \text{for } i = 1, \\
(x_{i+1} - x_{i-2}) \cdot x_{i-1} - x_i + F & \text{for } 2 \leq i \leq n - 1, \\
(x_1 - x_{n-2}) \cdot x_{n-1} - x_n + F & \text{for } i = n,
\end{cases}
\]

where \( x_i \) stands for the \( i \)-th model component, for \( 1 \leq i \leq n \).

Each model component stands for a particle which fluctuates in the atmosphere.

Exhibits chaotic behaviour when the external force \( F \) is set to 8.
The Lorenz 96 Model - Toy Model II

(a) $x_5$

(b) $x_{10}$

(c) $x_{20}$

(d) $x_{30}$

(e) $x_{35}$

(f) $x_{40}$

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Estimation of $\mathbf{B}$ via $N = 10^5$.

![Figure: Estimation of $\mathbf{B}$ via $N = 10^5$.](image)

(a) Structure

(b) Surf

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The Stochastic Ensemble Kalman Filter [Eve06] I

- Sequential Monte Carlo method for parameter and state estimation.

- Analysis ensemble (posterior ensemble):

\[
X^a = X^b + P^b \cdot H^T \cdot \left[ R + H \cdot P^b \cdot H \right] \cdot D
\]

\[
X^a = X^b + P^a \cdot H^T \cdot R^{-1} D \in \mathbb{R}^{n \times N},
\]

\[
X^a = P^a \cdot \left[ H^T \cdot R^{-1} \cdot Y_s + \left[ P^b \right]^{-1} \cdot X^b \right] \in \mathbb{R}^{n \times N},
\]

where \( P^a = \left[ H^T \cdot R^{-1} \cdot H + \left[ P^b \right]^{-1} \right] \in \mathbb{R}^{n \times n} \), and the \( e \)-th column of \( D \in \mathbb{R}^{m \times N} \) and \( Y_s \in \mathbb{R}^{n \times N} \) are:

\[
d[e] = y + \epsilon[e] - \mathcal{H} \left( x^{b[e]} \right) \in \mathbb{R}^{m \times 1}, \quad \text{and} \quad y^{s[e]} = y + \epsilon[e],
\]

respectively, for \( 1 \leq e \leq N \), and \( \epsilon[e] \sim \mathcal{N} \left( 0_m, \mathbf{R} \right) \).

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\[ L - 2 \text{ Error Norms in Time, } N = 10^5 \]

**Figure:** \( L - 2 \) error norms in time, \( N = 10^5 \).

**But too many samples!!!** In practice, model realizations are constrained by the hundreds...

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$L - 2$ error norms in time, $N = 10$

Figure: $L - 2$ error norms in time, $N = 10$.

What is going on here?...

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Estimation of $\mathbf{B}$ via $N = 10$

(a) Structure

(b) Surf

Figure: Estimation of $\mathbf{B}$ via $N = 10$.

What can we do? Localization methods...

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Localization Methods

- Avoid the impact of spurious correlations.
- If

\[
\frac{\log(n)}{N}
\]

is bounded (and small)... the resulting estimator is well-conditioned.

- Three different flavors:
  1. Covariance Matrix Localization. (Precision Localization) [NRSD15, NRSD17, NR17, NRSD18].
  2. Spatial Domain Localization [OHS$^+$04].
  3. Observation Localization [AND07, AND09].
Covariance Matrix Localization

- Impose the desired structure on $P^b$ via a decorrelation matrix.

$$\hat{P} = L \otimes P^b,$$

(2)

where, for instance,

$$\{L\}_{i,j} = \exp\left(-\frac{\phi(i,j)^2}{r^2}\right).$$

(a) $r = 1$
(b) $r = 3$
(c) $r = 5$
Effects of Covariance Matrix Localization

(a) $P^b$, $N = 30$
(b) $L$ for $r = 3$
(c) $\hat{P} = L \cdot P^b$

(d) $P^b$, $N = 30$
(e) $L$ for $r = 5$
(f) $\hat{P} = L \cdot P^b$

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$L - 2$ error norms in time.

(a) $N = 30$, $r = 1$, $p = 100\%$

(b) $N = 30$, $r = 3$, $p = 100\%$

(c) $N = 30$, $r = 5$, $p = 100\%$

(d) $N = 30$, $r = 1$, $p = 50\%$

(e) $N = 30$, $r = 3$, $p = 50\%$

(f) $N = 30$, $r = 5$, $p = 50\%$

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Component-wise products are prohibitive in high-dimensional spaces.

When two model components are conditional independent, their corresponding entry in the precision covariance matrix is zero.

(a) $r = 0$

(b) $r = 1$

(c) $r = 3$
Precision Matrix Localization II

- **Modified Cholesky Decomposition:**

\[
\hat{B}^{-1} = T^T \cdot D^{-1} \cdot T
\]

where the non-zero elements from \( T \in \mathbb{R}^{n \times n} \) are given by fitting models of the form:

\[
x[i] = \sum_{q \in P(i,r)} x[q] \cdot \{-T\}_{i,q} + \epsilon[i] \in \mathbb{R}^{N \times 1}, \text{ for } 1 \leq i \leq n,
\]

and \( \{D\}_{i,i} = \text{var}(\epsilon[i]) \).

\[\begin{array}{cccc}
1 & 5 & 9 & 13 \\
2 & 6 & 10 & 14 \\
3 & 7 & 11 & 15 \\
4 & 8 & 12 & 16 \\
\end{array}\]

\[\begin{array}{cccc}
1 & 5 & 9 & 13 \\
2 & 6 & 10 & 14 \\
3 & 7 & 11 & 15 \\
4 & 8 & 12 & 16 \\
\end{array}\]

(a) \( N(6, 1) \) (b) \( P(6, 1) \)

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An estimate:

(a) $P^b$  

(b) $T$  

(c) $D$  

(d) $\hat{B}^{-1} \text{ Str}$  

(e) $\hat{B}^{-1}$  

(f) $\hat{B}$  

Results:

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Precision Matrix Localization IV

(a) $N = 30$, $r = 1$, $p = 100$

(b) $N = 30$, $r = 3$, $p = 100$

(c) $N = 30$, $r = 5$, $p = 100$

(d) $N = 30$, $r = 1$, $p = 50$

(e) $N = 30$, $r = 3$, $p = 50$

(f) $N = 30$, $r = 5$, $p = 50$
Spatial Domain Localization [Bue11] I

Very simple idea:

(a) $r = 0$

(b) $r = 1$

(c) $r = 3$

Then...

1. Use local observations.
2. Use local estimators of covariance matrices.
3. Hybrid methods work very well.
4. Evidently, we mitigate the impact of sampling errors...
Spatial Domain Localization [Bue11] II

(a) $N = 30$, $r = 1$, $p = 100$

(b) $N = 30$, $r = 3$, $p = 100$

(c) $N = 30$, $r = 5$, $p = 100$

(d) $N = 30$, $r = 1$, $p = 50$

(e) $N = 30$, $r = 3$, $p = 50$

(f) $N = 30$, $r = 5$, $p = 50$

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Shrinkage Covariance Matrix Estimation

- Samples \( \{s_i\}_{i=1}^N \), where \( s_i \sim \mathcal{N}(0_n, C) \)
- Structure of matrices:
  \[
  \hat{C} = \gamma \cdot T + (1 - \gamma) \cdot C_s \in \mathbb{R}^{n \times n},
  \]
  optimal value of \( \gamma \) in squared loss sense
  \[
  \mathbb{E} \left[ \| \hat{C} - C \|_F^2 \right]
  \]
  where \( C \in \mathbb{R}^{n \times n} \) is the true covariance matrix. \( T = \frac{\text{tr}(C_s)}{n} \cdot I \).

- Properties:
  - Have been proven more accurate than the sample covariance matrix [CM14].
  - Better conditioned than the true covariance matrix [CWEH10].
  - They are strong under the condition \( n \gg N \) [CWH11].
Shrinkage Covariance Matrix Estimation II

▶ Ledoit and Wolf estimator [LW04, CWEH10]:

\[
\gamma_{LW} = \min \left( \frac{\sum_{i=1}^{N} \| C_s - s_i \otimes s_i^T \|_F^2}{N^2 \cdot \left[ \text{tr} \left( C_s^2 \right) - \frac{\text{tr}^2(C_s)}{n} \right]}, 1 \right)
\]

▶ Rao-Blackwell Ledoit and Wolf estimator [CWEH10]:

\[
\gamma_{RBLW} = \min \left( \frac{\frac{N-2}{n} \cdot \text{tr} \left( C_s^2 \right) + \text{tr}^2(C_s)}{(N + 2) \cdot \left[ \text{tr} \left( C_s^2 \right) - \frac{\text{tr}^2(C_s)}{n} \right]}, 1 \right)
\]

▶ It is proven that [CWH11]:

\[
\mathbb{E} \left[ \| \hat{C}_{RBLW} - C \|_F^2 \right] \leq \mathbb{E} \left[ \| \hat{C}_{LW} - C \|_F^2 \right].
\]
RBLW in the EnKF context

▶ Replace $P^b$ by a better estimator of $B$.

▶ RBLW estimator in the EnKF context:

$$\hat{B} = \gamma_{\hat{B}} \cdot [\mu_{\hat{B}} \cdot I_{n \times n}] + (1 - \gamma_{\hat{B}}) \cdot \hat{\delta}X \cdot \hat{\delta}X^T \in \mathbb{R}^{n \times n}.$$ 

where $\hat{\delta}X = \frac{1}{\sqrt{N-1}} \cdot \delta X \in \mathbb{R}^{n \times N}$.

▶ Parameters:

$$\mu_{\hat{B}} = \frac{\text{tr} \left( P^b \right)}{n}$$

$$\gamma_{\hat{B}} = \min \left( \frac{n^-2 \cdot \text{tr} \left( P^b^2 \right) + \text{tr}^2 \left( P^b \right)}{(N + 2) \cdot \left[ \text{tr} \left( P^b^2 \right) - \frac{\text{tr}^2(P^b)}{n} \right] \right), 1 \right)$$

▶ The direct implementation is prohibitive, recall $n \sim \mathcal{O}(10^8)$.

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Efficient Implementation of the RBLW I

Recall:

\[
\text{tr} \left( P^b \right) = \sum_{i=1}^{n} \sigma_i = \sum_{i=1}^{N-1} \sigma_i
\]

\[
\text{tr} \left( P^{b^2} \right) = \sum_{i=1}^{n} \sigma_i^2 = \sum_{i=1}^{N-1} \sigma_i^2
\]

Note

\[
P^b = \delta \hat{X} \cdot \delta \hat{X}^T = \left[ U_{\delta \hat{X}} \cdot \hat{\Sigma}_{\delta \hat{X}} \cdot V_{\delta \hat{X}}^T \right] \cdot \left[ U_{\delta \hat{X}} \cdot \hat{\Sigma}_{\delta \hat{X}} \cdot V_{\delta \hat{X}}^T \right]^T
\]

\[
= U_{\delta \hat{X}} \cdot \hat{\Sigma}^2_{\delta \hat{X}} \cdot U_{\delta \hat{X}}^T
\]

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Efficient Implementation of the RBLW II

this implies

\[ \sigma_i \left( P^b \right) = \hat{\sigma}_i^2 \left( \hat{\delta}X \right), \]

for \( 1 \leq i \leq N - 1. \)

▶ The estimator reads:

\[ \hat{B} = \gamma_B \cdot [\mu_{\hat{B}} \cdot I_{n \times n}] + (1 - \gamma_B) \cdot \hat{\delta}X \cdot \hat{\delta}X^T \in \mathbb{R}^{n \times n}. \]

▶ Efficient computation of the parameters:

\[ \mu_{\hat{B}} = \frac{\sum_{i=1}^{N-1} \hat{\sigma}_i^2}{n}, \]

\[ \gamma_B = \min \left( \frac{\frac{N-2}{n} \cdot \sum_{i=1}^{N-1} \hat{\sigma}_i^4 + \left[ \sum_{i=1}^{N-1} \hat{\sigma}_i^2 \right]^2}{(N + 2) \cdot \left[ \sum_{i=1}^{N-1} \hat{\sigma}_i^4 - \frac{\left[ \sum_{i=1}^{N-1} \hat{\sigma}_i^2 \right]^2}{n} \right]}, 1 \right). \]

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Efficient Implementation of the RBLW III

$\hat{\sigma}_i$ is the $i$-th singular value of $\delta X \in \mathbb{R}^{n \times N}$, for $1 \leq i \leq N - 1$. 
EnKF model space, with \( \varphi = \mu \hat{\gamma}_B \) and \( \delta = 1 - \gamma_B \):

\[
X^a = X^b + E \cdot \Pi \cdot Z_B + \varphi \cdot H^T \cdot Z_B,
\]

where \( E = \sqrt{\delta} \cdot \hat{\delta}X \in \mathbb{R}^{n \times N} \), \( \Pi = H \cdot E \in \mathbb{R}^{m \times N} \), and \( Z_B \in \mathbb{R}^{m \times N} \):

\[
\left( \Gamma + \Pi \cdot \Pi^T \right) \cdot Z_B = \begin{bmatrix} Y - H(X^b) \end{bmatrix},
\]

\[
\Gamma = R + \varphi \cdot H \cdot H^T \in \mathbb{R}^{m \times m}.
\]

EnKF ensemble space:

\[
X^a = X^b + U \cdot \lambda^* \in \mathbb{R}^{n \times N}.
\]

where \( U = \sqrt{N-1} \cdot \hat{\delta}X \in \mathbb{R}^{n \times N} \) and \( \lambda^* \in \mathbb{R}^{N \times N} \) minimizes

\[
J_{\text{ens}}(\lambda) = \frac{1}{2} \cdot \|U \cdot \lambda\|_{B-1}^2 + \frac{1}{2} \cdot \left\| Y - H(X^b) - Q \cdot \lambda \right\|_{R^{-1}}^2
\]

with \( Q = H \cdot U \in \mathbb{R}^{m \times N} \).
Synthetic Members

- The size of the ensemble can be increased by synthetic members:

\[ x_i^s \sim \mathcal{N}(\bar{x}^b, \hat{\mathcal{B}}), \text{ for } 1 \leq i \leq K. \]

- Sampling from the above distribution does not require to build \( \hat{\mathcal{B}} \), instead:

\[ \hat{\mathcal{B}} \equiv [\hat{\delta X}, \hat{\mu_B}, \hat{\gamma_B}] \]

- Prior distributions:

(a) \( K = 0 \)

(b) \( K = 120 \)
Sampling in High Dimensions I

- Taking the samples

\[
x_i^b = \bar{x}^b + \hat{B}^{1/2} \cdot \xi_i = \bar{x}^b + \left( \varphi \cdot I_{n \times n} + \delta \cdot \hat{\delta} \cdot \hat{\delta}^T \right)^{1/2} \cdot \xi_i
\]

where \( \xi_i \sim \mathcal{N} (0_n, I_{n \times n}) \), \( \varphi = \mu_{\hat{B}} \cdot \gamma_{\hat{B}} \) and \( \delta = 1 - \gamma_{\hat{B}} \).

- Consider the random vectors

\[
\begin{align*}
\xi_i^1 & \sim \mathcal{N} (0_n, I_{n \times n}) \in \mathbb{R}^{n \times 1}, \\
\xi_i^2 & \sim \mathcal{N} (0_N, I_{N \times N}) \in \mathbb{R}^{N \times 1},
\end{align*}
\]

and let

\[
\begin{align*}
\text{Cov} (\xi_i^1, \xi_i^2) &= \xi_i^1 \otimes \xi_i^2^T = 0_{n \times N}, \\
\text{Cov} (\xi_2, \xi_1) &= \xi_i^2 \otimes \xi_i^1^T = 0_{N \times n}.
\end{align*}
\]
We make the following substitution:

\[ \hat{B}^{1/2} \cdot \xi_i \sim \sqrt{\varphi} \cdot \xi_i^1 + \sqrt{\delta} \cdot \hat{\delta}X \cdot \xi_i^2. \]
The statistics are not changed:

\[
\mathbb{E} \left[ \left( \sqrt{\varphi} \cdot \xi_i^1 + \sqrt{\delta} \cdot \widehat{\delta X} \cdot \xi_i^2 \right) \left( \sqrt{\varphi} \cdot \xi_i^1 + \sqrt{\delta} \cdot \widehat{\delta X} \cdot \xi_i^2 \right)^T \right]
\]

\[
= \varphi \cdot \xi_i^1 \otimes \xi_i^1^T + \sqrt{\varphi} \cdot \delta \cdot \xi_i^1 \otimes \xi_i^2^T + \sqrt{\varphi} \cdot \delta \cdot \xi_i^2 \otimes \xi_i^1^T + \delta \cdot \widehat{\delta X} \cdot \xi_i^2 \otimes \xi_i^2^T \cdot \widehat{\delta X}^T
\]

\[
\text{Cov}(\xi_i^1, \xi_i^1) = I_{n \times n}, \quad \text{Cov}(\xi_i^1, \xi_i^2) = 0_{n \times N}, \quad \text{Cov}(\xi_i^2, \xi_i^1) = 0_{N \times n}, \quad \text{Cov}(\xi_i^2, \xi_i^2) = I_{N \times N}
\]

\[
= \widehat{B}.
\]
Sampling in High Dimensions IV

The synthetic members are obtained as follows:

\[ x^s_i = \bar{x}^b + \sqrt{\varphi} \cdot \xi^1_i + \sqrt{\delta} \cdot \hat{\delta} \cdot \hat{X} \cdot \xi^2_i, \quad i = 1, \ldots, K. \]
Importance of Synthetic Members
EnKF-MC and EnKF-SC with the SPEEDY Model I

- We make use of FORTRAN 90 in order to code the EnKF-MC and the EnKF-RBLW (from now on EnKF-SC).
- 96 ensemble members were used for the experiments.
- The initial perturbation of the background state is 5% the true state of the system.
- The model is propagated for a period of 24 days, observations are taken every 2 days.
- The SPEEDY model is used with T-63 resolution (96 \times 192) with 4 variables. 8 layers per variable. \( n \approx 590,000 \).
- Three sparse observational networks were used for the tests.
- We compare the results with the LETKF [OHS+04, BT99].
EnKF-MC and EnKF-SC with the SPEEDY Model II

Figure: Observational networks for different values of $p$. 

(d) $p = 12\%$

(e) $p = 6\%$

(f) $p = 4\%$

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Accuracy of the EnKF-MC I

Figure: RMSE of the LETKF and EnKF-MC implementations for different model variables, radii of influence and observational networks.

(a) \( r = 3 \) and \( p = 12\% \)

(b) \( r = 5 \) and \( p = 6\% \)

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Accuracy of the EnKF-MC II

Figure: 5-th layer of the meridional wind component ($v$).
Accuracy of the EnKF-MC III

(a) Reference  
(b) Background

(c) EnKF-MC  
(d) LETKF

Figure: 2-th layer of the zonal wind component ($u$).
Local Estimation of $B^{-1}$

(a) $T$

(b) $\hat{B}^{-1}$

(c) $\hat{B}$

(d) $\hat{B}$

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**Accuracy of the EnKF-RBLW I**

**(e) \( r = 3 \) and \( p = 12\% \)**

**(f) \( r = 5 \) and \( p = 6\% \)**

**Figure:** RMSE of the LETKF and EnKF-RBLW implementations for different model variables, radii of influence and observational networks.
Accuracy of the EnKF-RBLW II

(a) Reference  
(b) Background

(c) EnKF-RBLW  
(d) LETKF

**Figure:** 5-th layer of the meridional wind component ($v$).
Accuracy of the EnKF-RBLW III

Figure: 2-th layer of the zonal wind component ($u$).
Parallel implementations of ensemble based methods

- **Blueridge Super Computer @ VT**
  - BlueRidge is a 408-node Cray CS-300 cluster.
  - Each node is outfitted with two octa-core Intel Sandy Bridge CPUs and 64 GB of memory.
  - Total of 6,528 cores and 27.3 TB of memory systemwide.
  - Eighteen nodes have 128 GB of memory.
  - In addition, 130 nodes are outfitted with two Intel MIC (Xeon Phi) coprocessors.

- The methods are coded in FORTRAN using MPI.

- LAPACK \([\text{ABD}^+ 90]\) and BLAS \([\text{BDD}^+ 01]\) are used in order to efficiently perform matrix computations.

- **We vary the number of processors from 96 (16 computing nodes) to 2,048 (128 computing nodes)**
Parallel implementations of ensemble based methods I

- The approximations are based on **domain decomposition**

(a) 12

(b) 80

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Parallel implementations of ensemble based methods II

- Boundary information
Parallel implementations of ensemble based methods III

- Accuracy (EnKF-MC): number of processors ranges from 96 (16 computing nodes) to 2,048 (128 computing nodes)
Accuracy (EnKF-MC): number of processors ranges from 96 (16 computing nodes) to 2,048 (128 computing nodes)
Parallel implementations of ensemble based methods V

- Accuracy (EnKF-RBLW): number of processors ranges from 96 (16 computing nodes) to 2,048 (128 computing nodes)
Accuracy (EnKF-RBLW): number of processors ranges from 96 (16 computing nodes) to 2,048 (128 computing nodes)
Computational time: number of processors ranges from 96 (16 computing nodes) to 2,048 (128 computing nodes)
EnKF-MC Publications


EnKF-SC Publications


Bibliography I


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