Breaking Say's Law in a Simple Market Economy Model

(Draft)

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Introduction

A long tradition of development economics that comes from Adam Smith (1776) has sustained the importance of both division of labour (productive diversification) and extent of the market (population purchasing power) as fundamental engines of economic growth. According to Smith, their interaction brings about the circular causation of income: the sequence of productive diversification (technological change), exploitation of scale economies and expansion of the population's purchasing power increases products and incomes so that this economic cycle resumes at a higher level. Hence, productive diversification and effective demand push the process of economic growth in a virtuous circle in spiral form that goes from supply to demand and vice versa. Although Smith puts the supply effect (technological change and productive diversification) as the spark that ignites the growth process, he does not neglect the necessary supporting role of effective demand. In his view both supply and demand concur in a related but relatively independent way to the economic growth process. Thus, issues like a limited market size or a high concentration of income might slow down the economic activity and even stop it. In fact, according to Smith, the economic process cannot unfold if the expansion of supply, which is driven by a higher productivity, is not matched by an increased effective demand that can only come from economic activities where the distribution of remunerations to productive factors is sufficiently homogeneous. It is in that sense that Smith claims that the extent of the market limits the division of labour (Smith, 1776, book 1, ch. 3).

After the imposition of the neoclassical school as the main stream of economic thinking, most economic analysts ended up believing one way or another that demand follows supply, as the famous Say's law proposed. The criticism of many economic thinkers to this so-called law (Malthus, 1820; Marx, 1867, 1885 and 1894; Young,

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1928; Keynes, 1936; Rosenstein-Rodan, 1943; Nurkse, 1953; Currie, 1981; Murphy, Shleifer and Vishny, 1989a and 1989b) has been ignored or at least neglected. To be fair, it has to be said that the Post Keynesian economic school has sustained consistently the proposition that effective demand impacts economic growth (Harrod, 1936; Domar, 1946; Dutt, 2003). Notwithstanding, probably due to the huge impact of Keynes on short-term macroeconomics analysis, the role of effective demand is usually thought of as an issue of the (short-run) economic activity level. For the long-run period the conventional wisdom is that demand follows supply, and that is it. This is not surprising if one takes into account that most neoclassical models of economic growth ignore the impact of effective demand on long-run growth. These models usually consider consumers as if they were endowed with homothetic and non-satiable preferences. Under this assumption income distribution becomes irrelevant because consumers always behave in the same way regardless of their income level (Murphy et al., 1989b). Thus, economic models with those types of preferences obey Say's law if it is assumed that prices are flexible, transaction costs are negligible and markets clear.

Murphy et al. (1989a and 1989b) emphasized on the impact of industrialization and the population purchasing power on economic growth. The second paper in particular builds a model where the market size and the distribution of income play a central role in the explanation of economic activity. However, due to the difficulty of the subject, they focus the analysis on the proof of the existence of economic equilibrium in a general setting that embodies the interaction among capitalists, landowners and workers. The added value of this paper is then to build a simpler mathematically tractable model in which the interaction between capitalists and workers highlights the role of capital distribution (and income distribution) as a fundamental feature of the aggregate level of economic activity. In order to do that preferences are assumed to be non-homothetic (there exists a basic level of food consumption), and consumption of manufacturing goods is supposed to be satiable (none or just one unit of each available manufacturing good is consumed per person). The model, as it will

be shown below, is not free of mathematical complexity. In any case numerical results are attainable through the use of simple electronic spreadsheets.

A warning is in order. The important feature of product diversification as a mechanism for enhancing productivity is not included in this model in order to keep the economic system as simple as possible. Hence, the paper is unable to address the important issue of economic growth. Instead the effect of wealth distribution (and income distribution) on the economic activity level is emphasized. Besides, as is well-known, the productive effect of product diversification has been extensively analysed in well-known models of economic growth.

The Model

Consider a market economy where only food and manufactures are produced. Food is taken as *numeraire*. The food technology is given by the following Leontief production function: $F/\alpha = \min(K_F, L_F)$, where F is the food production in the period of analysis, α is a given level of multifactor productivity, and K_F and L_F are the amounts of capital and labour used in food production. The cost function associated to this technology under competitive conditions is given by $C(F) = [(r+w)/\alpha]F$, where r is the user cost of capital, and w is the wage. Given the linearity of the cost function, the profit function of the food sector is also linear in F: $\pi_F = F - [(r+w)/\alpha]F$. Note that food is taken as the *numeraire*. In a competitive environment with flexible prices and free mobility of productive factors profits are driven to nil. Hence, the factor price frontier is given by:

$$(1) \qquad w + r = \alpha.$$

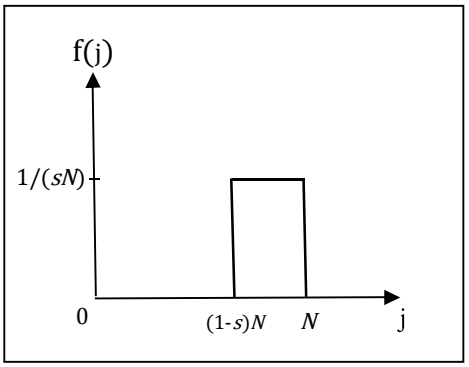
The Leontief technology embodies perfect complementarity between the factors. A more general technological specification, like a CES production function, would model factor substitutability at the cost of greater mathematical entanglement. In any case, the factor price frontier yields, as always, an opposing relationship between capitalists and workers at the distribution level.

Given the food technology, the factor demands in the agricultural sector are defined by:

(2) and (3) $K_F = L_F = F/\alpha$.

Capital and population, K and N, are given at any production period. Labour is homogeneously distributed among the N members of this economy (one unit *per capita*). For mathematical simplicity of the model it is assumed that property of capital is homogeneously distributed among capitalists, who represent a fraction s of the society. Capital property concentration is thought of as the inherited result of a historic process of capital accumulation. It is also assumed that no one can be a worker and a capitalist at the same time; these roles are exclusive. Hence, at any production period there exist sN capitalists and $L = (1-s)N$ workers. The Figure 1 depicts the distribution function of capital property for the N members of the society ordered by j according to their capital rents (first workers and then capitalists).

Figure 1
Capital Property Distribution Function



Source: the author.

Labour and capital are assumed to be inelastically supplied at non-negative remunerations.

Let us consider now the typical worker consumption problem. Preferences are given by the following utility function: $Z(f_W, m) = \ln(f_W - \gamma) + \theta\ln(m)$, where f_W is the amount of food consumed in the period of analysis by the typical worker (and all workers are alike), γ is the minimum amount of food that a person needs during the production period, m is the diversification index of manufacturing goods for workers, and θ is a positive number that measures the bias of the consumer towards

manufacturing consumption.[2] The marginal utility of food decreases with its own consumption. Only one or none of each manufacturing good is consumed; and the marginal utility of manufacturing goods decreases steadily with their own diversification, m. The typical worker budget restriction is given by the following expression: $f_W + pm = w$, where p is the price of the manufactured goods which are consumed by workers. As it will be shown below, the demand and technology of all manufactured goods which are demanded by workers are the same across goods. Hence they have a single common price. The solution of this optimization problem yields the following demand functions:

$$(4) \qquad pm = [\theta/(1+\theta)](w - \gamma)$$

And

$$f_W = (w + \theta\gamma)/(1+\theta).$$

Hence, the typical worker consumes manufacturing goods if and only if the wage covers at least the basic food demand ($m \geq 0$ if and only if $w \geq \gamma$).

Capitalists consume all the manufacturing goods which are demanded by workers, m, plus some others, $M–m$. The manufactures of general use, those indexed between 0 and m, will be called from now on high-demand goods. Hence, for a typical capitalist the consumption problem is set as follows: maximize $Z(f_K, M) = \ln(f_K - \gamma) + \theta\ln(M)$ subject to the following budget restriction: $f_K + pm + P(M - m) = rk_e$, where f_K is the amount of food consumed in the period of analysis by the typical capitalist (and all capitalists are alike), M is the typical capitalist index of manufacturing consumption diversification, P is the common price of all those manufacturing goods which are demanded only by capitalists –they will be called from now on the low-demand goods, the difference $M - m$ is the range of low-demand manufacturing goods, and k_e is the amount of effective capital per capitalist [as it will be shown below, capital might be unemployed, hence $k_e \leq k \equiv K/(sN)$]. The typical capitalist budget restriction means that he/she consumes food and high-demand manufacturing goods as any typical worker does, but also consumes some other manufacturing goods. As in the case of

[2] It could be important and realistic to consider explicitly some manufactures as basic goods. This option is excluded in order to keep the model as simple as possible.

high-demand goods, supply and demand of low-demand goods are assumed to be identical across these goods; that is why they all have the same price. The solution of this optimization problem yields the following demand functions:

(5) $PM = [\theta/(1+\theta)][rk_e + (P - p)m - \gamma]$

And

$$f_K = [rk_e + (P - p)m + \theta\gamma]/(1+\theta).$$

Notice that a sensible general equilibrium for a market economy must imply that the typical capitalist remuneration, rk_e, is higher than the wage, w. Otherwise every capitalist would prefer to become a worker, and the capitalist society would implode.

Effective aggregate capital demand is given by

(6) $K_e = sNk_e$,

where K_e is the fraction of total capital which is effectively used in economic activities. In a market economy this would imply that capital shares yield a lower return because some capital fraction, $K - K_e$, might be unemployed.

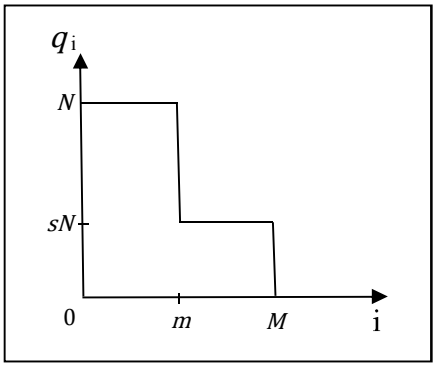
Food demand has two components: workers´ demand and capitalists´ demand. Thus, aggregate food demand is given by $F = (1-s)Nf_W + sNf_K$. By substituting in the previous expression the demands for food from a typical worker, f_W, and from a typical capitalists, f_K, yields the aggregate food demand:

(7) $F = [\theta\gamma + (1-s)w + srk_e + s(P-p)m]N/(1+\theta)$.

Manufacturing goods are all produced with the following technology: a fixed investment of ϕ units of capital is needed in order to create a new manufacturing good. Besides β units of labour are required to produce one manufacturing good. This cost structure implies a technology with increasing returns to scale.

Taking into account that each person in this economy consumes one or none of each manufacturing good, the demand structure for the whole range of manufacturing goods is shown by Figure 2.

Figure 2
Demand Structure for Manufacturing Goods



Source: the author.

In this figure q_i represents the society´s consumption of the i-th manufacturing good. Every individual consumes one unit of all high-demand goods ($0 < i \leq m$); and every capitalist consumes one unit of all low-demand goods ($m < i \leq M$). The total demand of high-demand goods is given by Nm; and the total demand of low-demand goods is given by $sN(M-m)$. Thus, aggregate profits of high-demand manufacturing firms are given by

$$\pi = (Nm)p - (Nm)\beta w - \phi mr.$$

And aggregate profits of low-demand goods are given by

$$\Pi = [sN(M-m)]P - [sN(M-m)]\beta w - \phi(M-m)r.$$

The market structure of manufacturing goods in this model resembles the case of monopolistic competition; in this case, however, prices and consumptions are predetermined. Since market entry and exit are assumed to be free, prices should be set for all manufacturing goods such that profits are null everywhere: $\pi = \Pi = 0$. Hence, the equilibrium prices of high-demand manufacturing goods and low-demand manufacturing goods, p and P, should satisfy the following equations:

(8) $p = \beta w + \phi r/N$ for $0 < i \leq m$

and

(9) $P = \beta w + \phi r/(sN)$ for $m < i \leq M$.

These equilibrium conditions imply that prices of low-demand manufacturing goods are higher than prices of high-demand manufacturing goods: $P > p$. This outcome is

explained by the fact that a common fixed investment per sector implies higher average fixed costs for low-demand goods than for high-demand goods.

Because of the consumption rigidities this economic system does not guarantee equilibrium in the factor markets. In any case aggregate capital demand, K_e, cannot exceed capital supply, K:

$$K_F + \phi M = K_e \leq K. \quad (10)$$

And, on the other hand, aggregate labour demand cannot exceed labour supply:

$$L_F + \beta Nm + \beta sN(M-m) \leq L = (1-s)N. \quad (11)$$

All parameters denoted with Greek letters are assumed to be constant: the agricultural multifactor productivity (α), the marginal cost of manufactures (β), the fixed cost of manufactures (ϕ), the basic food requirement per capita (γ), and the fraction θ of the utility function. The population (N), the capital stock (K), and the population fraction of capitalists (s) are also given. Hence, the economic system has eleven unknown variables: w, r, F, K_F, L_F, p, P, m, M, k_e and K_e. With nine equations and two inequations there exists the possibility of a general equilibrium if the slackness is supressed (the labour market and the capital markets clear). Unfortunately the solutions are mathematically intricate. Numerical solutions may be found, however, by means of a simple spreadsheet package.

Finding Solutions

The economic system has some degree of recursivity from the mathematical viewpoint. Taking into account that the wage is limited by the agricultural productivity ($0 \leq w \leq \alpha$), the arbitrary choice of w within this interval determines immediately, by equation (1), the user cost of capital, r. Given w and r, the prices p and P, for high-demand and low-demand manufacturing goods, are determined by equations (8) and (9). Given w and p, the diversity of manufacturing consumption by a typical worker, m, is determined by equation (4). The combination of equations (5), (6) and (10) yields solutions for the effective capital demand per capitalist (k_e), the aggregate effective capital demand (K_e), and the diversity of manufacturing

consumption by a typical capitalist (M).[3] The key variable here is the capital demand per capitalist, k_e, since, in any case, it cannot overcome the available capital per capitalist: i.e. $k_e \leq k$, where $k \equiv K/(sN)$. Whenever capital demand exceeds capital supply, the last variable, i.e. k, is used in the system. Now, given the previously mentioned variables aggregate food demand, F, is solved by equation (7), and factor demands in the agricultural sector, K_F and L_F, are also solved by equations (2) and (3). Finally, gross domestic product is defined as follows: $GDP = F + pNm + PsN(M-m)$.

First case: Extent of the Market Constraint

In the first case that is examined below, the general economic equilibrium is solved for the following specific parameters: $\alpha = 2$; $\beta = 1{,}3$; $\gamma = 0{,}5$; $\phi = 2$; and $\theta = 0{,}5$. It is also assumed that the capital stock and the population are given by $K = 30$ and $N = 30$. It is assumed that concentration of capital property is given by $s = 0{,}2$: only one of every five persons is a capitalist. As Table 1 shows, there is no wage between the basic subsistence remuneration (the wage that just buys the minimum food demand: $\gamma = 0{,}5$) and the maximum wage ($\alpha = 2$) that yields full capital employment ($UK > 0$).[4] Labour supply is fully employed ($UL = 0$).[5] Notice that the factor price adjustment does not work: for a lower capital user costs than $r = 0{,}541$ (higher wage than $w = 1{,}459$) the typical capital remuneration falls below the wage: $rk_e/w < 1$. Moreover, in this particular case, even the minimum user cost of capital ($r = 0{,}01$), or the maximum wage ($w = 1{,}99$), does not clear the capital market: effective capital demand is always lower than capital supply. Therefore, the lack of aggregate demand due to the low population size prevents an economic equilibrium. The industrial economy cannot take off, capital profitability collapses, and all factors are assigned completely to the agricultural sector. With a constant returns to scale technology, the agricultural sector may accommodate the productive factors if they come in the right proportions (1 to 1

[3] The mentioned combination yields the effective capital per capitalist as a function of the previously solved variables:

$k_e = \{[(1-s)w+\theta\gamma+s(P-p)m]/[\alpha(1+\theta)]+[\theta/(1+\theta)][(P-p)m-\gamma](1/P)(\phi/N)\}/\{[s-sr/[\alpha(1+\theta)]-[\theta/(1+\theta)](r/P)(\phi/N)]\}$

An equivalent mathematical expression with identical solution is found by combining the inequation (11) as an equality (the slackness is supressed) with equation (5).

[4] Capital unemployment is defined by $UK = K - (K_F + \phi M)$.

[5] Labour unemployment is defined by $UL = (1-s)N - [L_F + \beta Nm + \beta sN(M-m)]$.

in this case); but the increasing returns to scale technologies of the manufacturing sector requires a minimum demand level in order to cover fixed costs.[6]

Table 1
Extent of the Market Constraint
Parameters: $\alpha = 2$; $\beta = 1{,}3$; $\gamma = 0{,}5$; $\phi = 2$; $\theta = 0{,}5$; $K = 30$; $N = 30$.

s	w	r	p	P	m	k_e		k	M	Ke	rk_e/w	F	$K_F = L_F$	GDP	UK	UL
0,2	0,500	1,500	0,750	1,150	0,000	**2,714**	<	**5**	1,331	16,28	8,141	27,24	13,62	36,42	**13,718**	0
0,2	0,550	1,450	0,812	1,198	0,026	**2,686**	<	**5**	1,218	16,11	7,081	27,36	13,68	36,57	**13,886**	0
0,2	0,600	1,400	0,873	1,247	0,049	**2,665**	<	**5**	1,117	15,99	6,218	27,51	13,76	36,79	**14,010**	0
0,2	0,650	1,350	0,935	1,295	0,069	**2,650**	<	**5**	1,027	15,90	5,504	27,69	13,85	37,07	**14,100**	0
0,2	0,700	1,300	0,997	1,343	0,086	**2,640**	<	**5**	0,945	15,84	4,902	27,90	13,95	37,39	**14,163**	0
0,2	0,750	1,250	1,058	1,392	0,101	**2,633**	<	**5**	0,870	15,80	4,388	28,11	14,06	37,75	**14,204**	0
0,2	0,800	1,200	1,120	1,440	0,115	**2,629**	<	**5**	0,801	15,77	3,943	28,34	14,17	38,13	**14,227**	0
0,2	0,850	1,150	1,182	1,488	0,127	**2,627**	<	**5**	0,737	15,76	3,555	28,58	14,29	38,53	**14,236**	0
0,2	0,900	1,100	1,243	1,537	0,138	**2,628**	<	**5**	0,678	15,77	3,212	28,82	14,41	38,94	**14,234**	0
0,2	0,950	1,050	1,305	1,585	0,148	**2,630**	<	**5**	0,623	15,78	2,906	29,07	14,53	39,37	**14,222**	0
0,2	1,000	1,000	1,367	1,633	0,157	**2,633**	<	**5**	0,571	15,80	2,633	29,31	14,66	39,8	**14,202**	0
0,2	1,050	0,950	1,428	1,682	0,165	**2,637**	<	**5**	0,522	15,83	2,386	29,56	14,78	40,23	**14,175**	0
0,2	1,100	0,900	1,490	1,730	0,173	**2,643**	<	**5**	0,476	15,86	2,162	29,81	14,91	40,67	**14,143**	0
0,2	1,150	0,850	1,552	1,778	0,180	**2,649**	<	**5**	0,432	15,89	1,958	30,06	15,03	41,11	**14,107**	0
0,2	1,200	0,800	1,613	1,827	0,186	**2,656**	<	**5**	0,390	15,93	1,770	30,31	15,15	41,55	**14,066**	0
0,2	1,250	0,750	1,675	1,875	0,192	**2,663**	<	**5**	0,351	15,98	1,598	30,55	15,28	41,98	**14,023**	0
0,2	1,300	0,700	1,737	1,923	0,197	**2,671**	<	**5**	0,313	16,02	1,438	30,79	15,4	42,42	**13,977**	0
0,2	1,350	0,650	1,798	1,972	0,203	**2,679**	<	**5**	0,277	16,07	1,290	31,03	15,52	42,85	**13,929**	0
0,2	1,400	0,600	1,860	2,020	0,207	**2,687**	<	**5**	0,243	16,12	1,151	31,27	15,63	43,27	**13,879**	0
0,2	**1,459**	**0,541**	1,933	2,077	0,213	**2,697**	<	**5**	0,204	16,18	**1,000**	31,55	15,77	43,77	**13,819**	0
0,2	1,500	0,500	1,983	2,117	0,216	**2,704**	<	**5**	0,178	16,22	**0,901**	31,73	15,87	44,11	**13,776**	0
0,2	1,550	0,450	2,045	2,165	0,220	**2,713**	<	**5**	0,148	16,28	**0,788**	31,96	15,98	44,52	**13,723**	0
0,2	1,600	0,400	2,107	2,213	0,224	**2,722**	<	**5**	0,119	16,33	**0,680**	32,19	16,09	44,93	**13,670**	0
0,2	1,650	0,350	2,168	2,262	0,227	**2,731**	<	**5**	0,090	16,38	**0,579**	32,41	16,2	45,33	**13,616**	0
0,2	1,700	0,300	2,230	2,310	0,231	**2,740**	<	**5**	0,063	16,44	**0,483**	32,62	16,31	45,73	**13,562**	0
0,2	1,750	0,250	2,292	2,358	0,234	**2,749**	<	**5**	0,037	16,49	**0,393**	32,84	16,42	46,12	**13,507**	0
0,2	1,800	0,200	2,353	2,407	0,237	**2,758**	<	**5**	0,011	16,55	**0,306**	33,05	16,52	46,51	**13,453**	0
0,2	1,850	0,150	2,415	2,455	0,240	**2,767**	<	**5**	-0,013	16,60	**0,224**	33,26	16,63	46,89	**13,398**	0
0,2	1,900	0,100	2,477	2,503	0,242	**2,776**	<	**5**	-0,037	16,66	**0,146**	33,46	16,73	47,27	**13,344**	0
0,2	1,950	0,050	2,538	2,552	0,245	**2,785**	<	**5**	-0,060	16,71	**0,071**	33,66	16,83	47,64	**13,290**	0
0,2	1,990	0,010	2,588	2,590	0,247	**2,792**	<	**5**	-0,078	16,75	**0,014**	33,82	16,91	47,93	**13,247**	0

Source: the author.

Second Case: Capital Concentration Constraint

The analysis moves on to consider different levels of capital concentration under the assumption of market clearing for goods and factors. In this new case population is tripled: $N = 60$, but all other parameters are set just as before. Table 2 shows the numerical results. With a higher market demand there appear some price remunerations for labour and capital that determine a general economic equilibrium. As a matter of fact, when capital concentration is given by $s = 20\%$, as in the previous analysis, a general economic equilibrium is achieved with the following couple of factor prices: $w = 1{,}011$ and $r = 0{,}989$ (see the Table). Notwithstanding, general

[6] "As it is the power of exchanging that gives occasion to the division of labour, so the extent of this division must always be limited by the extent of that power, or, in other words, by the extent of the market. When the market is very small, no person can have any encouragement to dedicate himself entirely to one employment (…). There are some sorts of industry, even of the lowest kind, which can be carried on no where but a great town" [Smith 1776, I, III (1981, I, p. 31)].

equilibrium is not guaranteed since capital concentration (and income concentration) might become a relevant constraint for economic activity.

Table 2
Capital Concentration Constraint
Parameters: $\alpha = 2$; $\beta = 1{,}3$; $\gamma = 0{,}5$; $\phi = 2$; $\theta = 0{,}5$; $K = 30$; $N = 60$.

	s	w	r	p	P	m	k_e		k	M	K_e	rk_e/w	F	$KF = LF$	GDP	UK	UL
	0,33	1,990	0,010	2,587	2,588	0,247	1,475	<	1,515	-0,080	29,21	**0,007**	58,739	29,37	80,29	0,79	0
	0,32	1,990	0,010	2,587	2,588	0,247	1,539	<	1,563	-0,080	29,55	**0,008**	59,423	29,71	81,49	0,45	0
	0,31	1,990	0,010	2,587	2,588	0,247	1,607	<	1,613	-0,080	29,89	**0,008**	60,108	30,05	82,68	0,11	0
	0,30	1,907	0,093	2,482	2,489	0,243	1,667	=	1,667	-0,059	30	**0,081**	60,237	30,12	82,88	0	0
1	0,29	1,794	0,206	2,340	2,356	0,237	1,724	=	1,724	-0,026	30	**0,197**	60,103	30,05	82,61	0	0
	0,28	1,691	0,309	2,208	2,235	0,231	1,786	=	1,786	0,011	30	**0,327**	59,954	29,98	82,32	0	0
	0,27	1,594	0,406	2,086	2,123	0,225	1,852	=	1,852	0,052	30	**0,471**	59,791	29,9	82	0	0
	0,26	1,503	0,497	1,971	2,018	0,218	1,923	=	1,923	0,099	30	**0,635**	59,605	29,8	81,65	0	0
	0,25	1,417	0,583	1,861	1,920	0,211	2,000	=	2,000	0,151	30	**0,823**	59,394	29,7	81,25	0	0
	0,24	1,334	0,666	1,757	1,827	0,204	2,083	=	2,083	0,211	30	1,039	59,155	29,58	80,82	**0**	**0**
	0,23	1,254	0,746	1,655	1,738	0,195	2,174	=	2,174	0,281	30	1,294	58,877	29,44	80,31	**0**	**0**
	0,22	1,174	0,826	1,554	1,652	0,186	2,273	=	2,273	0,362	30	1,598	58,553	29,28	79,73	**0**	**0**
	0,21	1,094	0,906	1,453	1,566	0,175	2,381	=	2,381	0,459	30	1,971	58,165	29,08	79,04	**0**	**0**
2	0,20	**1,011**	**0,989**	1,347	1,479	0,163	2,500	=	2,500	0,578	30	2,446	57,689	28,84	78,2	**0**	**0**
	0,19	0,921	1,079	1,234	1,387	0,146	2,632	=	2,632	0,729	30	3,080	57,084	28,54	77,14	**0**	**0**
	0,18	0,817	1,183	1,101	1,281	0,123	2,778	=	2,778	0,940	30	4,025	56,24	28,12	75,68	**0**	**0**
	0,17	0,662	1,338	0,905	1,123	0,077	2,941	=	2,941	1,317	30	5,945	54,73	27,37	73,11	**0**	**0**
	0,16	0,6	1,4	0,827	1,072	0,052	**3,171**	>	**3,125**	1,555	30	7,292	54,207	27,1	72,24	**0**	**0,497**
	0,15	0,6	1,4	0,827	1,091	0,052	**3,447**	>	**3,333**	1,642	30	7,778	54,413	27,21	72,6	**0**	**1,145**
	0,14	0,6	1,4	0,827	1,113	0,052	**3,770**	>	**3,571**	1,738	30	8,333	54,62	27,31	72,96	**0**	**1,834**
	0,13	0,6	1,4	0,827	1,139	0,052	**4,150**	>	**3,846**	1,844	30	8,974	54,826	27,41	73,32	**0**	**2,570**
	0,12	0,6	1,4	0,827	1,169	0,052	**4,604**	>	**4,167**	1,962	30	9,722	55,033	27,52	73,68	**0**	**3,361**
	0,11	0,6	1,4	0,827	1,204	0,052	**5,155**	>	**4,545**	2,094	30	10,606	55,24	27,62	74,04	**0**	**4,217**
	0,10	0,6	1,4	0,827	1,247	0,052	**5,837**	>	**5,000**	2,242	30	11,667	55,446	27,72	74,4	**0**	**5,150**
3	0,09	0,6	1,4	0,827	1,299	0,052	**6,698**	>	**5,556**	2,410	30	12,963	55,653	27,83	74,76	**0**	**6,175**
	0,08	0,6	1,4	0,827	1,363	0,052	**7,817**	>	**6,250**	2,602	30	14,583	55,859	27,93	75,12	**0**	**7,313**
	0,07	0,6	1,4	0,827	1,447	0,052	**9,320**	>	**7,143**	2,824	30	16,667	56,066	28,03	75,48	**0**	**8,588**
	0,06	0,6	1,4	0,827	1,558	0,052	**11,429**	>	**8,333**	3,083	30	19,444	56,272	28,14	75,84	**0**	**10,036**
	0,05	0,6	1,4	0,827	1,713	0,052	**14,562**	>	**10,000**	3,388	30	23,333	56,479	28,24	76,2	**0**	**11,704**
	0,04	0,6	1,4	0,827	1,947	0,052	**19,608**	>	**12,500**	3,755	30	29,167	56,685	28,34	76,56	**0**	**13,658**
	0,03	0,6	1,4	0,827	2,336	0,052	**28,773**	>	**16,667**	4,204	30	38,889	56,892	28,45	76,92	**0**	**15,994**

Source: the author.

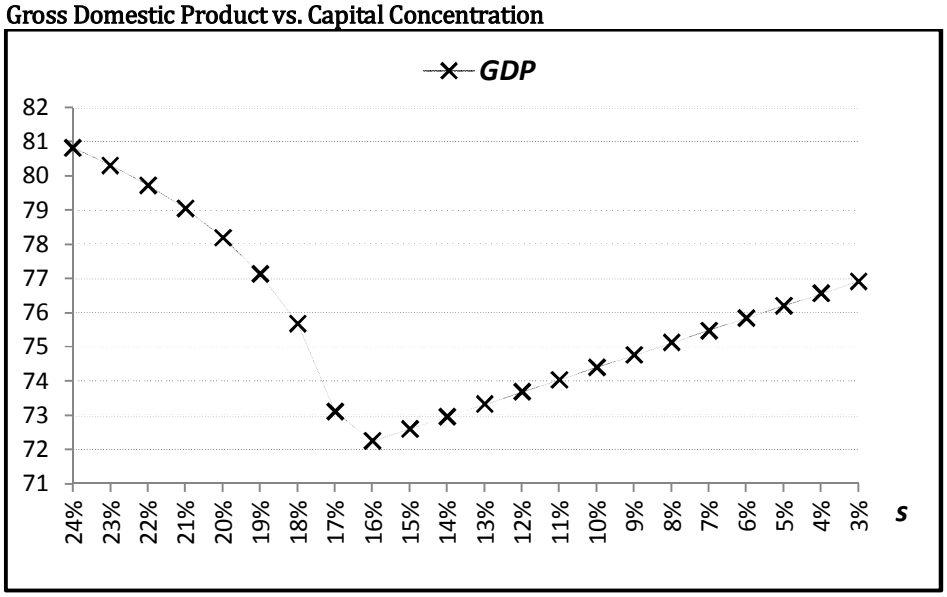
Table 2 focuses on the equilibrium results of the economy under a growing degree of capital property concentration (decreasing share of capitalist population). The Table delivers that this market economy requires some minimum level of capital concentration: for $s > 0{,}24$ (24%) the remuneration of the typical capitalist is lower than the wage: $rk_e/w < 1$. Hence, low levels of capital concentration, as in the first frame of Table 2, are not suitable for capitalist activity –an original capital accumulation process is unavoidable. Moreover, for very low levels of capital property concentration –as in the shadowed section in frame 1: $s \geq 29\%$- capital remuneration is so low that the model yields a negative consumption of manufactures by capitalists; although this is not a viable economic outcome it is reported here in order to exhibit the workings of the model. Now, the second frame of Table 2 reveals that full market equilibrium may occur for intermediate levels of capital concentration ($24\% \leq s \leq$

17%); in this circumstance unemployed labour and unemployed capital are nil: $UL = UK = 0$. Finally, the third frame of Table 2 shows that under a high degree of capital concentration ($s < 17\%$) the effective capital demand exceeds capital supply: $k_e > k$, and labour unemployment is unavoidable.

The price adjustment works for the frames 1 and 2 of Table 2. Under this circumstance (low and intermediate levels of capital concentration: $28\% \leq s \leq 17\%$) the economic logic is as follows: a higher capital concentration ($s \downarrow$) increases labour supply [L = (1-s)$N \uparrow$], concentrates capital and income in the hands of capitalists, lowers aggregate demand [GDP ↓], and the adjustment comes with a lower labour remuneration (w ↓), and, of course, a higher capital remuneration (r ↑). A lower wage is required in order to expand labour demand so that the labour market equilibrium is preserved. Now, under a high degree of capital concentration the working of the economic system is modified. All capital is fully employed, $UK = 0$, but there is always some labour unemployment. This last feature is due to the lack of aggregate demand. And the price adjustment does not work: lower wages just diminishes aggregate demand and increases unemployment; and higher wages –which would go some way in solving the problem by increasing aggregate demand- are not viable due to the excess of labour supply.[7] Thus, the price system of the labour market collapses under extreme capital concentration. Without government intervention wages would probably be set at the subsistence level ($w = \gamma = 0{,}5$) or even below, but the setting of some minimum wage above the subsistence wage helps to increase the level of economic activity. This feature is shown in the third frame of Table 2: when capital concentration is high ($s < 17\%$) the wage is fixed at $w = 0{,}6$ so that the workers might consume some manufacturing goods. Hence, as Graph 1 depicts, for every level of capital concentration the minimum wage increases the economic activity compared to the alternative of allowing the wage to fall freely.[8]

[7] These possible effects are not reported here but the author checked them all with the model.
[8] "No society can surely be flourishing and happy, of which the far greater part of the members are poor and miserable" [Smith 1776, I, VIII (1981, I, p. 70)].

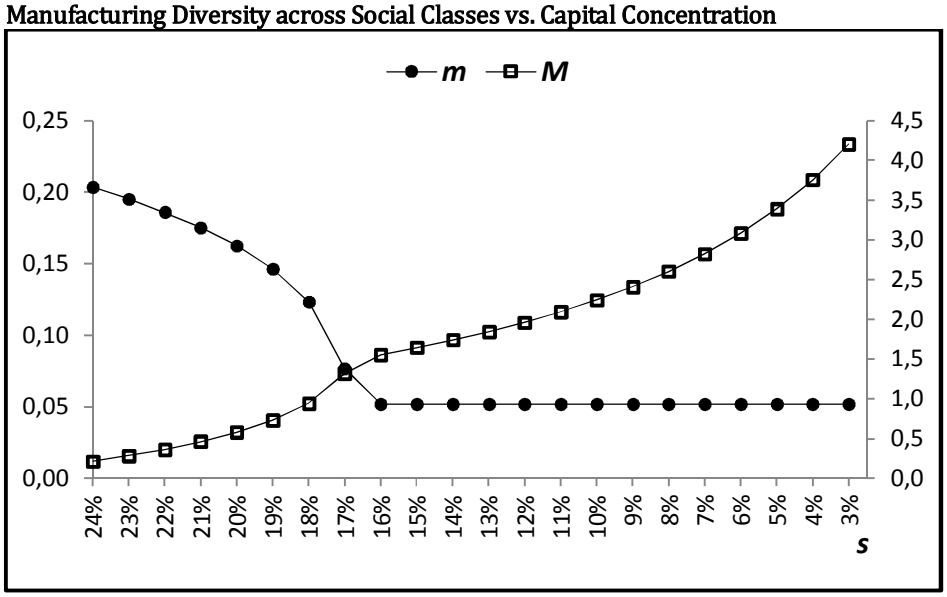
Graph 1
Gross Domestic Product vs. Capital Concentration



Source: the author.

Graph 1 also reveals that a higher degree of capital concentration diminishes the economic activity whilst the degree of capital concentration is consistent with general market equilibrium ($24\% \geq s \geq 17\%$); this result is the consequence of a fall of aggregate effective demand in the economy. When capital concentration becomes too high ($s < 17\%$), and the government enforces a minimum wage, *GDP* grows with the level of capital concentration. This result is explained by the expansion of capitalist consumption diversity (M increases) since the worker´s consumption diversity is unchanged. Graph 2 depicts these features. Therefore, setting a minimum wage under high capital concentration is not only favourable to workers but also to capitalists.

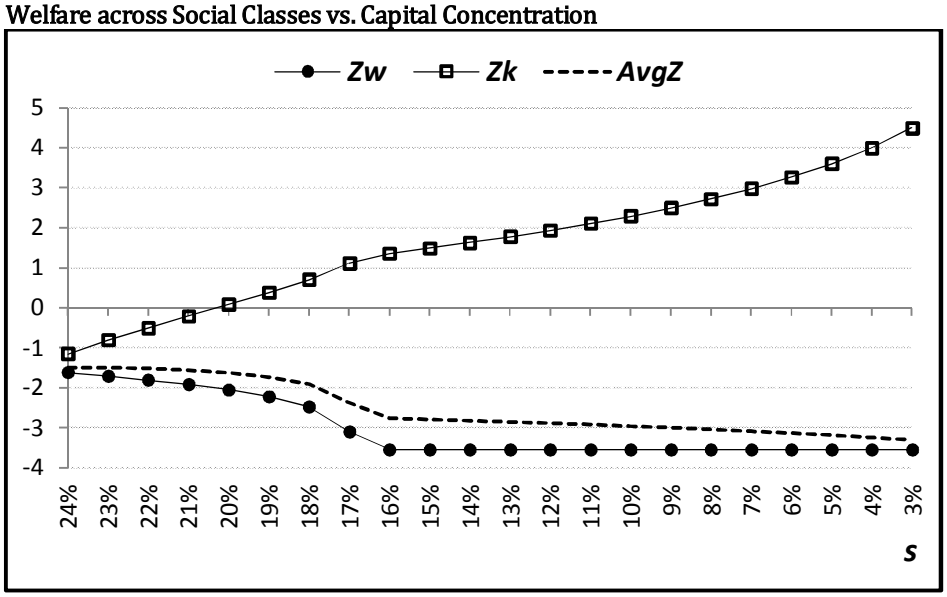
Graph 2
Manufacturing Diversity across Social Classes vs. Capital Concentration



Source: the author.

The evolution of welfare for a typical worker and a typical capitalist is depicted by Graph 3. As a general result, a growing degree of capital concentration worsens the welfare of workers, as measured with the utility function (Z_w), until the minimum wage is reached –under this circumstance a worker's welfare index remains constant. On the other hand, the welfare of the typical capitalist (Z_k) increases systematically with capital concentration. Since a higher capital concentration implies a growing fraction of population workers, the average welfare, *AvgZ*, which is depicted by the discontinuous line in Graph 3, decreases with capital concentration.

Graph 3
Welfare across Social Classes vs. Capital Concentration



Source: the author.

Although workers might find a low and stable welfare path when they earn the minimum wage, the subjective vision might be different because of the growing welfare gap between workers and capitalists in an environment with concentration of capital property.

Some Additional Results

Under circumstances of full market equilibrium (intermediate case which was examined above) and the assumption of *ceteris paribus*, a higher agricultural productivity (higher α) increases wages, improves labour diversity of manufacturing consumption, diminishes income inequality and expands strongly the gross domestic product. Table 3 exhibits these results.

Table 3
Higher Agricultural Productivity (α↑)

α	β	K	N	s	L	γ	θ	ϕ	w	r	p	P	m	k_e	k	M	K_e	rk_e/w	F	$K_F = L_F$	GDP	UK		UL
2	1,3	30	60	0,20	48	0,5	0,75	2	**1,0110**	0,9890	1,347	1,479	**0,163**	2,5	2,5	0,578	30	**2,45**	57,69	28,84	**78,2**	0		0
2,2	1,3	30	60	0,20	48	0,5	0,75	2	**1,3115**	0,8885	1,735	1,853	**0,201**	2,5	2,5	0,404	30	**1,69**	64,22	29,19	**89,6**	0		0

Source: the author.

Therefore, this model delivers that a country with unexploited land or low technological productivity might gain enormously by means of a land reform and heavy investment in science and technology applied to agriculture and agroindustrial activities.

Under circumstances of full market equilibrium and under the assumption of *ceteris paribus*, a higher mechanization of manufacturing activities, which is considered here with a lower β (less demand for labour in manufacturing activities), lowers wages, increases income inequality and lowers the gross domestic product. Table 4 shows these results.

Table 4
Manufacturing Mechanization (β↓)

α	β	K	N	s	L	γ	θ	ϕ	w	r	p	P	m	k_e	k	M	K_e	rk_e/w	UK	F	$K_F = L_F$	GDP	UL
2	**1,4**	30	60	0,20	48,0	0,5	0,75	2	**1,0378**	0,9622	1,485	1,613	0,155	2,5	2,5	0,511	30	**2,318**	0,0	57,954	28,98	**78,68**	0,0
2	**1**	30	60	0,20	48,0	0,5	0,75	2	**0,8456**	1,1544	0,884	1,038	0,168	2,5	2,5	0,996	30	**3,413**	0,0	56,018	28,01	**75,22**	0,0

Source: the author.

These results are indicative of the impacts of robotization on the economy. It is convenient to recall, however, that this model focuses on the demand side; the likely productive effects of economic diversification might modify the above conclusion.

Final Remarks

This is a static model and so it is unable to address the important issue of economic growth. Instead it focuses on the impact of the population purchasing power on the level of economic activity. The dynamic implications of this model for economic growth should be found in the transmission mechanism by which productive activity is transformed into incomes and market demands. To that extent this paper finds that a general economic equilibrium is available for intermediate levels of capital property concentration. Under this circumstance a growing level of wealth inequality induces a lower wage, increases income inequality and diminishes the *GDP*. For high levels of capital property concentration the labour market does not adjust and this feature justifies the setting of a minimum wage. The model is then consistent with the observed trend of income concentration under a seemingly unstoppable process of wealth concentration.

If, as some authors argue, the capitalist regime has an intrinsic trend toward capital property concentration (Marx, 1867, 1885 and 1894; Piketty, 2013), it seems sensible to assume that sooner or later the economy will reach the degree of capital property inequality such that unemployment is unavoidable. Presumably this is an environment prone to economic crisis. Notwithstanding, as Marx pointed out, some

minimum level of labour unemployment might be necessary in the capitalist regime in order to offset the wage pressures derived from a growing labour organization –this is the famous hypothesis of the industrial reserve army. Thus, some degree of excessive capital property concentration may be functional to the capitalist society.

It seems, however, that the trend towards capital concentration does not stop in sensible levels. On the contrary, it is generally accepted that wealth concentration has been increasing all over the world and this trend shows no sign of being reversed. Under extreme concentration of wealth, income redistribution and even property redistribution might be necessary in order to preserve the economic activity level and enhance social welfare. This model, however, does not support capital expropriation in the hands of the state. Actually, it finds that under some intermediate levels of capital concentration the economic system may operate with full market equilibrium –it might be the secret of the virtuous capitalist economies of northern Europe.

It would be desirable to put more structure in this model in order to address the issues of product diversification externalities, economic growth, land rents, short-run and long-run unemployment, poverty, international trade, etc. Hopefully this model may pave the way for future developments.

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