Braun Trees in Agda

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What are Braun trees?

Among the many types of balanced binary trees, the Braun trees (Braun & Rem, 1983) are perhaps the most limited. A Braun tree is a binary tree which is as balanced as it can possibly be, every node satisfies the following condition:

- The left subtree has either the same number of nodes as the right subtree or one more.

Braun trees

A binary tree is a Braun tree if:

- It is empty or
- Its left and right subtrees are Braun trees

Braun trees are balanced, their maximum depth is O ($\log_2$ n), where n is the number of elements in the tree.

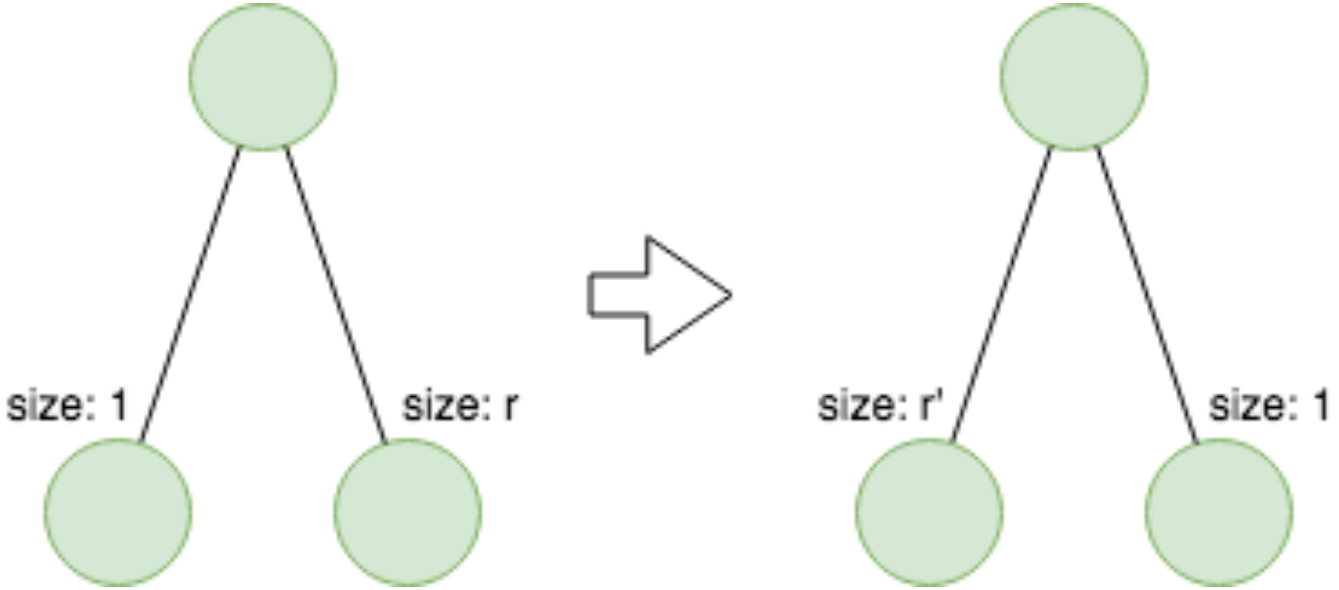
Why in Agda?

With dependent types (Stump, 2015) we can define the type of height-balanced trees of a certain size, the type BraunTree is indexed by a natural number which represents the size of the tree.

BraunTree 0
BraunTree 1
:
:
:
BraunTree n

Property balanced of Braun trees

The trick for maintain the property of balanced Braun trees occur during insertion data.



Data type Braun trees

The index n is the size of the tree (number of elements of type A)

```
postulate
  A    : Set
  _<A_ : A → A → 𝔹

data BraunTree : (n : ℕ) → Set where
  empty : BraunTree 0
  node  : ∀ {m n}
        → A → BraunTree m → BraunTree n
        → m ≡ n ∨ m ≡ suc n
        → BraunTree (suc (m + n))
```

Data type Braun trees

```
postulate
 a : A

data₁ : BraunTree 0
data₁ = empty

data₂ : BraunTree 1
data₂ = node a
                empty
                empty
                (inj₁ refl)
```

```
data₃ : BraunTree 2
data₃ = node a
                (node a
                  empty
                  empty
                  (inj₁ refl))
                empty
                (inj₂ refl)
```

Method of Braun trees

- Insert

```
{- we will keep smaller (_<A_) elements closer to the root of the Braun tree as we insert -}
btInsert : ∀ {n} → A → BraunTree n → BraunTree (suc n)
btInsert x empty = node x empty empty (inj₁ refl)
btInsert x (node{m}{n} y treeₗ treeᵣ p)
  rewrite +comm m n
  with    p         | if x <A y then (x , y) else (y , x)
...  | inj₁ m≡n    | (v₁ , v₂) = node v₁ (btInsert v₂ treeᵣ) treeₗ (inj₂ (cong suc (sym m≡n)))
...  | inj₂ m≡sucn | (v₁ , v₂) = node v₁ (btInsert v₂ treeᵣ) treeₗ (inj₁ (sym m≡sucn))
```

Method of Braun trees

- Insert

```
insert₁ : BraunTree 2
insert₁ = btInsert a
                   (btInsert a empty)

insert₂ : BraunTree 1
insert₂ = btInsert a
                   empty
```

```
insert₃ : BraunTree 3
insert₃ = btInsert a
                   data₃

insert₄ : BraunTree 2
insert₄ = btInsert a
                   data₂
```

Bibliography

- W. Braun. and M . Rem. (1983) A logarithmic implementation of flexible arrays. Memorandum MR83/4. Eindhoven University of Technology.
- A. Stump. (2015) Verified Functional Programing in Agda. [Online]. Disponible: https://play.google.com/books/reader?id=kMwvDAAAQBAJ&printsec=frontcover&output=reader&hl=es&pg=GBS.PP1

THANKS!