Modeling Colombian yields with a macro-factor affine term structure model

Research practise 3: Final report

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***Abstract*—We model and forecast the Colombian yield curve using affine term structure models with macroeconomic factors. Models with different combinations of macro-factors outperform purely latent models at in-sample fit and out-of-sample one month ahead forecasting. The improvements are most evident in the short end of the curve.**

I. Introduction

Interest rates vary over time and maturity (investment time horizon). The time varying relationship between interest rates and their maturities is commonly called the 'term structure of interest rates' (TS for short). The TS is partially observable through the yields at which different bonds of the same issuer are traded in the market.

The TS plays a central role in an economy. Current yields have useful information for forecasting future short yields and, potentially, real economic activity, inflation, and other key economic variables, Piazzesi (2010). Market participants use these forecasts for pricing financial assets, taking investment decisions, and managing financial risks. Central banks use them to inform monetary policy. Consumers use them to make saving and consumption decisions. Thus, superior modeling and forecasting of the yield curve serve policymakers in evaluating past, current, and future economic conditions and help market participants and consumers in taking better financial decisions.

Movements of the yield curve have traditionally been explained by three latent factors related to its level, slope, and curvature (Nelson and Siegel, 1987; Diebold and Li, 2006). However, including macroeconomic factors in models has been beneficial for interpretation (Diebold et al., 2006) and performance (Ang and Piazzesi, 2003). If the yield curve reacts to market conditions, including macroeconomic factors should enhance models' explanatory power and reveal information about the effects of these factors on the fixed income market.

In this study, we model and forecast the Colombian yield curve (1) using no-arbitrage affine term structure models (ATSMs) with latent and macroeconomic factors. We continue the line of study from Velásquez-Giraldo and Restrepo-Tobón (2016), in which we found a high difficulty in forecasting short yields using purely latent ATSMs. We explained this phenomenon with the hypothesis that short yields are formed as reactions to macroeconomic conditions, which we test in this article using macroeconomic factors and a different estimation methodology.

Using monthly data from 2005 to 2015, we estimate Gaussian ATSMs with three latent factors and two macroeconomic factors, testing 28 pairs of the latter. We use the estimation methodology presented in Hamilton and Wu (2012). We find that macro-factor models perform better at modelling and forecasting the yield curve when compared with a purely latent model. The model with the best in-sample fit uses the inter-bank

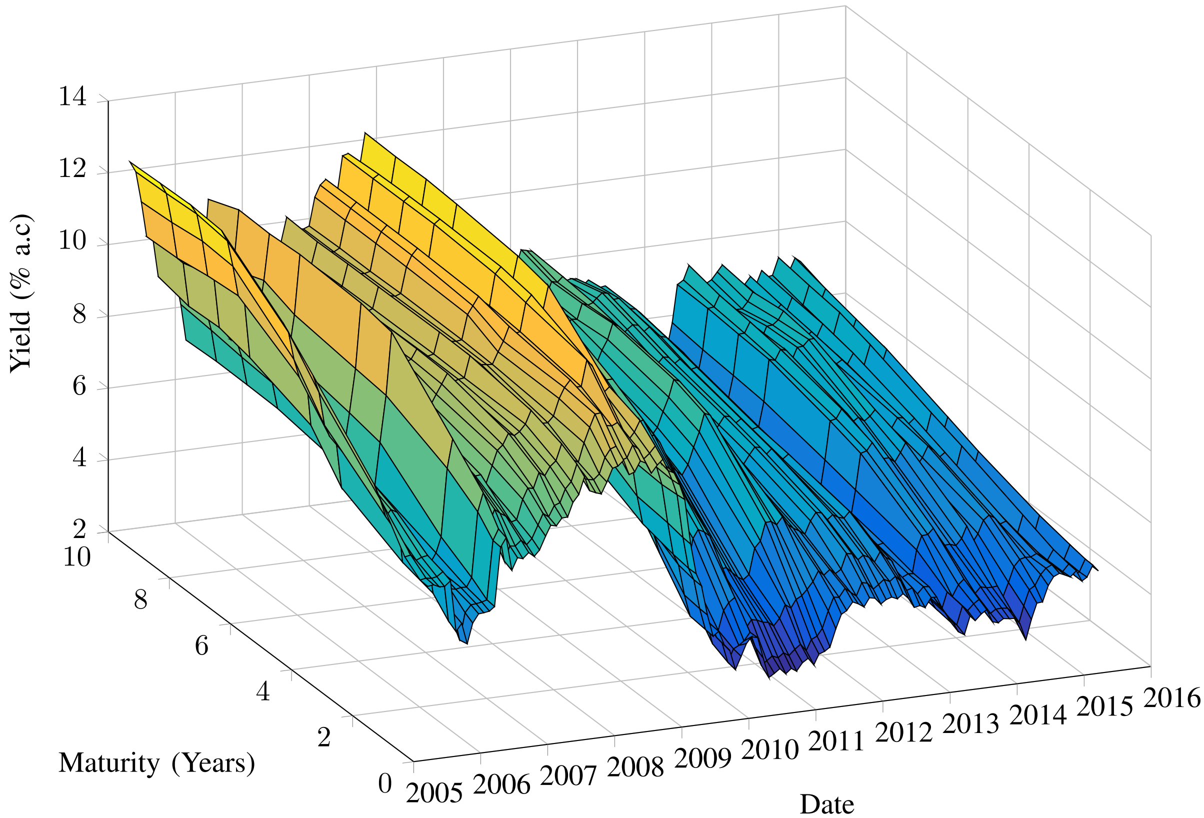

Figure 1. Historical behavior of the Colombian yield curve. Authors' elaboration using data from Bloomberg.

rate and the monetary base, while the best out-of-sample forecast are obtained using an index of economic activity and the monetary base. In both cases, the performance enhancement occurs at the short end of the curve.

The article is organized as follows. Section II presents ATSMs and the methodology that we adopt. Section III presents and discusses the main empirical results. Concluding remarks and future lines of work are presented in Section IV.

II. Methodology

Affine term structure models, first introduced by Duffie and Kan (1996), model the TS as an affine function[1] of a state vector. The state vector changes over time (t) and the affine function varies with maturity (τ), which gives ATSMs the required versatility to model yield curves.

We will part from a three-factor Gaussian ATSM, which is set up as follows (Piazzesi, 2010).

The state vector $X(t)$ is assumed to follow an affine diffusion process under the risk-neutral probability measure Q:

$$dX(t) = \mu^Q(X)dt + dW^Q(t)$$

where $\mu^Q : \mathbb{R}^3 \to \mathbb{R}^3$ is an affine function and W^Q is a 3-dimensional independent brownian motion under Q.

We denote the continuously compounded yield with maturity τ at time t by $\gamma_\tau(t)$. The short rate $r(t)$ is also assumed to be affine on $X(t)$:

$$r(t) = \lim_{\tau \to 0} \gamma_\tau(t) = \delta_0 + \delta_1^\top X(t)$$

where $\delta_0 \in \mathbb{R}$ and $\delta_1 \in \mathbb{R}^3$.

If the no-arbitrage hypothesis holds, the price of a pure discount bond with maturity τ at time t should be given by the following equation:

$$P(X(t), \tau) = E^Q\left[exp\left(-\int_t^{t+\tau} r(u)\,du\right)|X(t)\right]$$

where E^Q denotes the conditional expected value under Q.

Duffie and Kan (1996) show that for these assumptions to hold, $\gamma_\tau(t)$ must also be an affine functions of the state vector for every τ:

$$\gamma_\tau(t) = A(\tau) + B(\tau)^\top X(t) \tag{1}$$

[1] A function $F : \mathbb{R}^N \to \mathbb{R}^M$ is said to be affine if $F(X) = A + B * X$ for some vector A and matrix B.

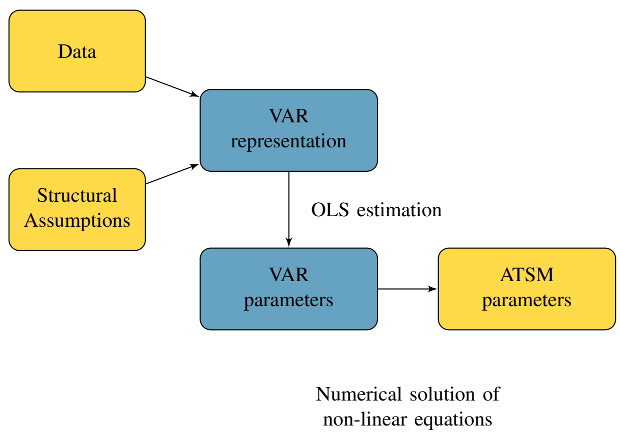

Figure 2. Summary of the estimation methodology proposed by Hamilton and Wu (2012).

where $A(\tau)$ and $B(\tau)$ are obtained from a set of ordinary differential equations that result from imposition of no-arbitrage restrictions.

To obtain the state dynamics under the physical probability measure P, the market price of risk $\Lambda(X(t)) : \mathbb{R}^3 \to \mathbb{R}^3$ must be defined. Depending on its specification, the state dynamics might or might not be affine under P:

$$dX(t) = (\mu^Q(X) + \Lambda(X))dt + dW^P(t)$$

For estimation of the models, we use the methodology proposed by Hamilton and Wu (2012). They show that discrete time Gaussian ATSMs, can be represented as vector auto-regressive models (VARs), which facilitates estimation. The procedure is summarized in Figure 2.

To keep the models identified (guaranteeing existence and uniqueness of the mapping between the ATSM and VAR structures), Hamilton and Wu (2012) restrict the number of yields and macro-factors in each type of model:

- Latent three-factor models include three yields observed without errors and one observed with Gaussian errors.
- Macro-factor models with three latent factors include two macro-factors, three yields observed without errors, and three yields observed with Gaussian errors.

We pick the three-months, five-years, and ten-years yields to be observed without errors for both specifications. For the latent factor model, the 1 year yield is picked as observed with errors. For macro-factor models, the one-year, four-year, and eight-year yields are used as observed with errors.

After estimation, models' fit and forecast accuracy is evaluated. The macro-factor models using all the combinations of factors are ranked according to their *root mean squared errors* (RMSEs) for both in-sample fit and one-month forecasts of yields for all maturities. We include additional maturities to evaluate the capacity of modelling yields that are not included in the estimation process.

We analyse the macroeconomic factor loadings ($B(\tau)$ in Equation 1) of the best models to gain insight into their effect on the different parts of the yield curve.

We work with a dataset of bootstrapped zero-coupon Colombian yields obtained from Bloomberg covering the time period between April 2005 and May 2015. We work with monthly observations. The macroeconomic variables taken into consideration are:

- IMACO: monthly index of economic activity
- IPC: consumer price index
- USD/COP exchange rate
- CDS 5y: 5 years credit default swap rate for Colombian sovereign bonds
- VIX: global volatility index from the Chicago Board Options Exchange Market
- WTI: oil price
- TIB-IBR: inter-bank interest rate, formed by joining two indicators (IBR and TIB) which were used in different time periods by Colombia's Central Bank
- Monetary base: amount of currency circulating in the economy, as measured by the Central Bank

All macro factors are obtained from Bloomberg and Colombia's Central Bank (Banco de la República de Colombia), and are included as monthly variations.

III. Results

We first estimate a purely latent factor model, finding that the numerical procedures involved converge quickly and consistently. Factor loadings for each factor are presented in Figure 3. We find similarities with the traditional level, slope, and curvature interpretations that are often attributed to these factors. However there are two main deviations from this usual interpretation: first, the factor which we label as 'level' has a decreasing effect (should be constant according to its interpretation), and the factor labeled as 'slope' has an initially decreasing effect for yields under one year (should be increasing for all maturities).

The factors are labelled based on comparisons with empirical proxies for the level, slope and curvature of the yield curve. Figure 4 shows that both the estimated level and curvature factors are very closely related to their empirical counterparts, while the same can not be said for the slope. We attribute the differences in the estimated slope behaviour to the abnormal shape of its

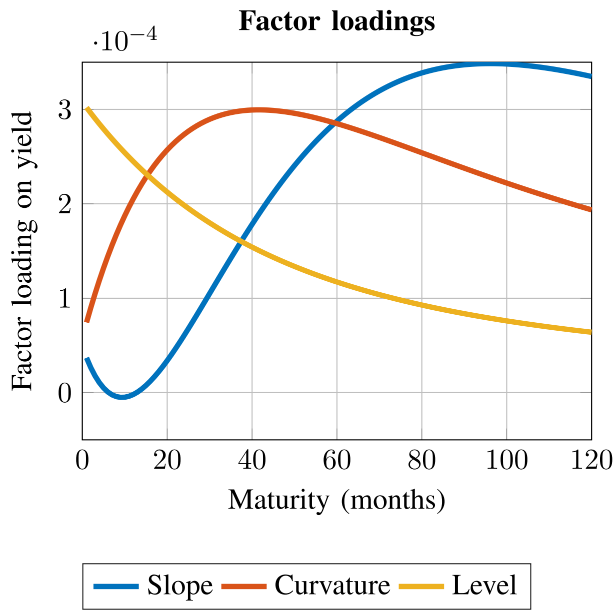

Figure 3. Estimated factor loadings for the latent factors in the latent factor model.

Figure 4. Estimated factors and their empirical counterparts. Values are standardized.

loadings depicted in Figure 3. If traditional labellings for the factors need to be strictly followed, factor rotation methodologies such as Procrustes analysis can be applied.

We now turn to macro-factor models. We estimate models with three latent and two macroeconomic factors using the 28 possible combinations of the factors from Section II. In-sample RMSEs for every model and maturity are reported in Table I. Errors for the latent factor model are also reported for comparison.

Various interesting results can be drawn from Table I. A first fact that can be highlighted is the explanatory power of the inter-bank rate (TIB-IBR). Five of the seven models in which it is included have the lowest average RMSEs. The inter-bank rate is an indicator that captures the rates at which banks are willing to lend and borrow money to each other over short periods of time. Therefore, it plays a determinant role in the short end of the yield curve, acting as a basis (or lower bound) upon which market yields are formed.

The inter-bank rate's role is confirmed upon inspection of its loadings over maturities. Figure 5 displays the loadings over maturity of the two macroeconomic factors in the model with the best fit from Table I (TIB-IBR and Monetary base). As expected, the inter-bank rate has a positive effect with a high magnitude for short yields, which diminishes as maturity goes up. The monetary base also has a positive effect which, although smaller in magnitude than the inter-bank rate's, steadily affects the short end of the curve. We interpret the positive effect of the monetary base of interest rates as a reduction of the value of money when its circulating amount raises.

With respect to the magnitude of errors, Table I shows that various macro-factor models manage to fit data better than the purely latent model, especially in the short end of the curve. Even for maturities not included in estimation (as six months and two years), the TIB-IBR - Monetary base model obtains RMSEs under half of those in the purely latent model. Figure 6 compares observed yields with those generated by the TIB-IBR - Monetary base model. The fit is very close across all maturities.

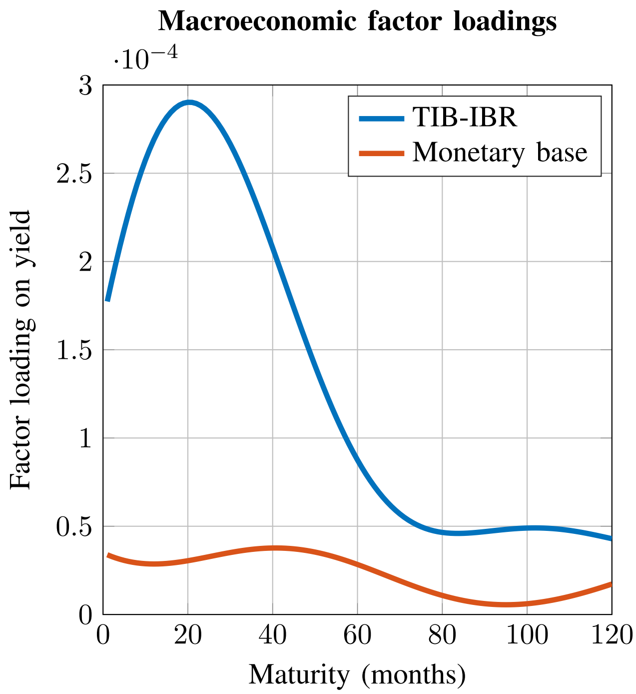

Figure 5. Factor loadings over maturity for the model with the best in-sample fit.

One month ahead forecasts are produced using every model over a validation set not used in estimation. Forecasted values are obtained as follows. First, we calculate latent factors at time t using yields observed without errors. With the latent factors and the (observed) macroeconomic factors at time t, we calculate their expected value at time $t+1$. Finally, we obtain the forecasted yields using the expected values for factors at $t+1$ and the factor loadings obtained form estimation.

Table II reports forecast errors for all possible combinations of macro-factor models and the purely latent model. Again, the best macro-factor models outperform the latent model. The comparison is specially favourable at the short end of the curve (yields with maturities up to two years), where RMSEs are considerably reduced by the inclusion of macroeconomic factors. The improvement on in-sample fit and forecast capabilities generated by the inclusion of macro factors is in line with results from literature (Ang and Piazzesi, 2003) which report that a big part of the variance in the short end of the curve is explained by macro-factors in no-arbitrage models.

It is important to note that models with the best in-sample fit (Table I) do not match those with the best forecasting capabilities (Table II). Particularly, models which include the TIB-IBR as a factor do not generate good forecasts. This result indicates that while its explanatory power is high, the inter-bank rate does not have a good predictive power. Reverse causality might be the cause of the forecasting inaccuracy, as the direction of the effect between market yields and the inter-bank rate is not clear. In the model with the best forecasts, the inter-bank rate is replaced by the IMACO, whereas the Monetary base keeps its place.

Figure 7 displays the factor loadings over maturities for the model with the best forecasts in Table II. The IMACO, an index of economic activity, has a positive relationship with short yields and a negative impact on yields with maturities over five years. A possible interpretation for this flattening effect can be an increase in demand for short-term credit due to high economic activity (raising short rates) and a decrease in the perceived risk (slightly lowering long rates). The Monetary base preserves its positive effect.

Figure 8 compares the forecasts generated with the IMACO-Monetary base model with the observed yields for all maturities. The accuracy of forecasts does not seem good in the considered period of time (May 2014 - April 2015), but the confidence intervals contain every real observation. Tests with more observations should be conducted, but data is scarce given its periodicity and the youth of the Colombian fixed-income market. However, monthly forecast RMSEs (Table II) have values around those reported in literature (Duffee, 2002; Ang and Piazzesi, 2003) and inferior to those achieved for Colombia in our previous study (Velásquez-Giraldo and Restrepo-Tobón, 2016).

IV. Concluding Remarks and Future Work

In this study, we model and forecast the Colombian yield curve using affine term structure models with various combinations of macroeconomic factors. We find that the inter-bank rate is very effective for in-sample modelling, having a big impact on the short end of the curve. For forecasting, an indicator of economic activity and the monetary base are the most effective combination. Economic activity has a flattening effect on the yield curve according to its factor loadings.

We find that including the right combinations of macroeconomic factors can improve the fitting and forecasting performance of ATSMs when compared with purely latent factor models. In accordance with past literature, the effect of the inclusion of macroeconomic factors is greater on the short end of the yield curve, supporting the idea that short yields from as reactions to economic conditions while long yields can mostly be explained by short yields.

The next step in our research line is to extend our results to other developing South American economies. We intend to find and compare the most influential driving factors of the yield curve for different countries.

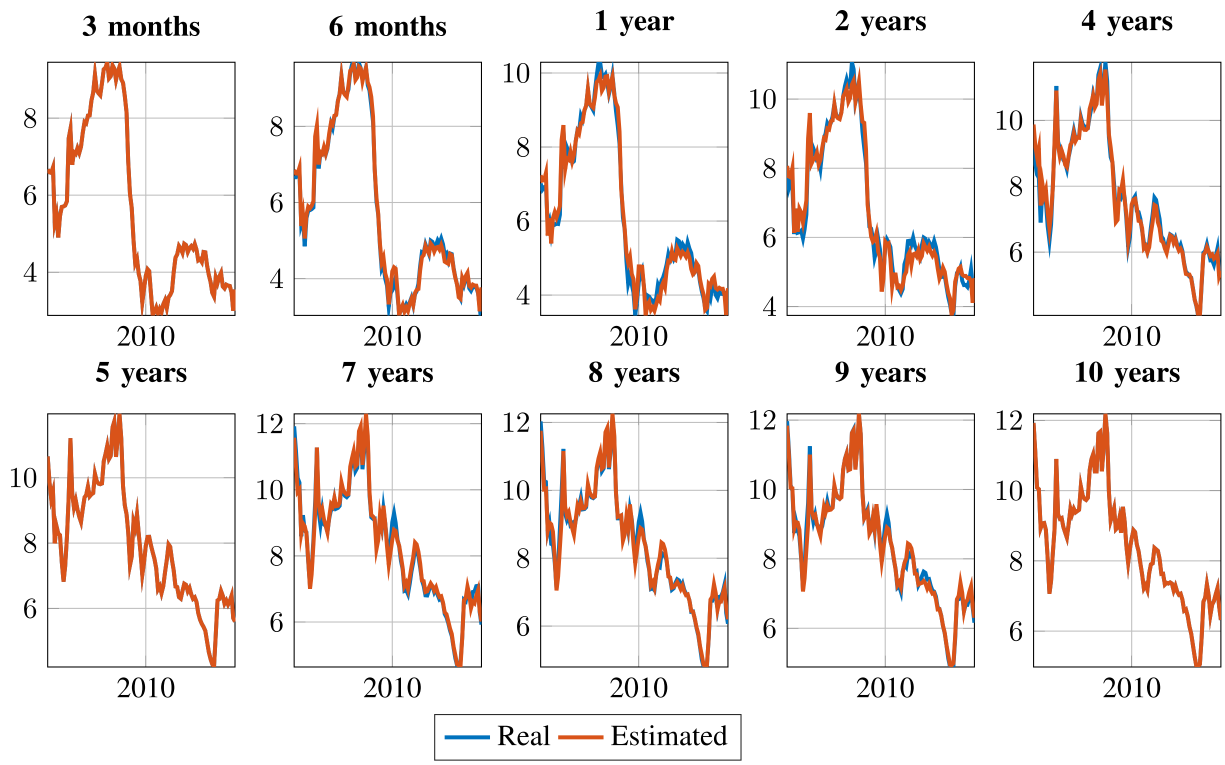

Figure 6. In-sample fit for all maturities with the IBR-Monetary base model. Yields are reported as continuously compounded with annual periodicity.

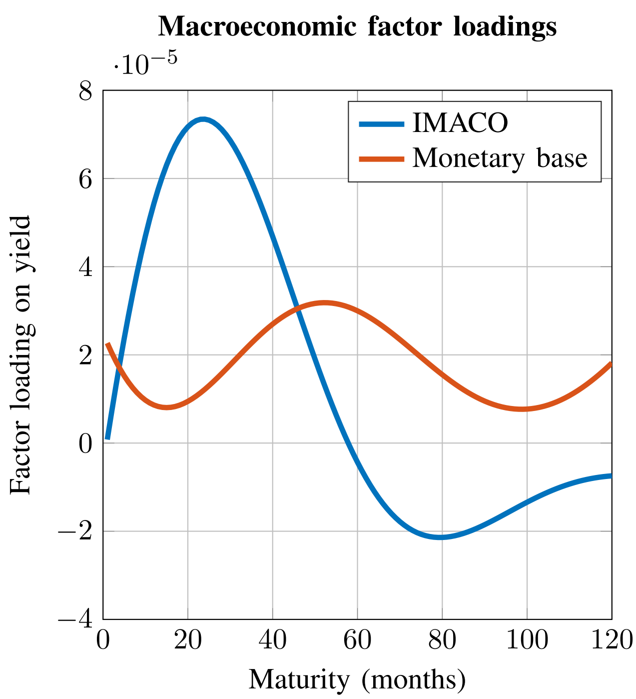

Figure 7. Factor loadings over maturity for the model with the best out of sample forecasts.

We believe this comparison could highlight interesting idiosyncratic characteristics and shed light onto traits of fixed income markets at different stages of development.

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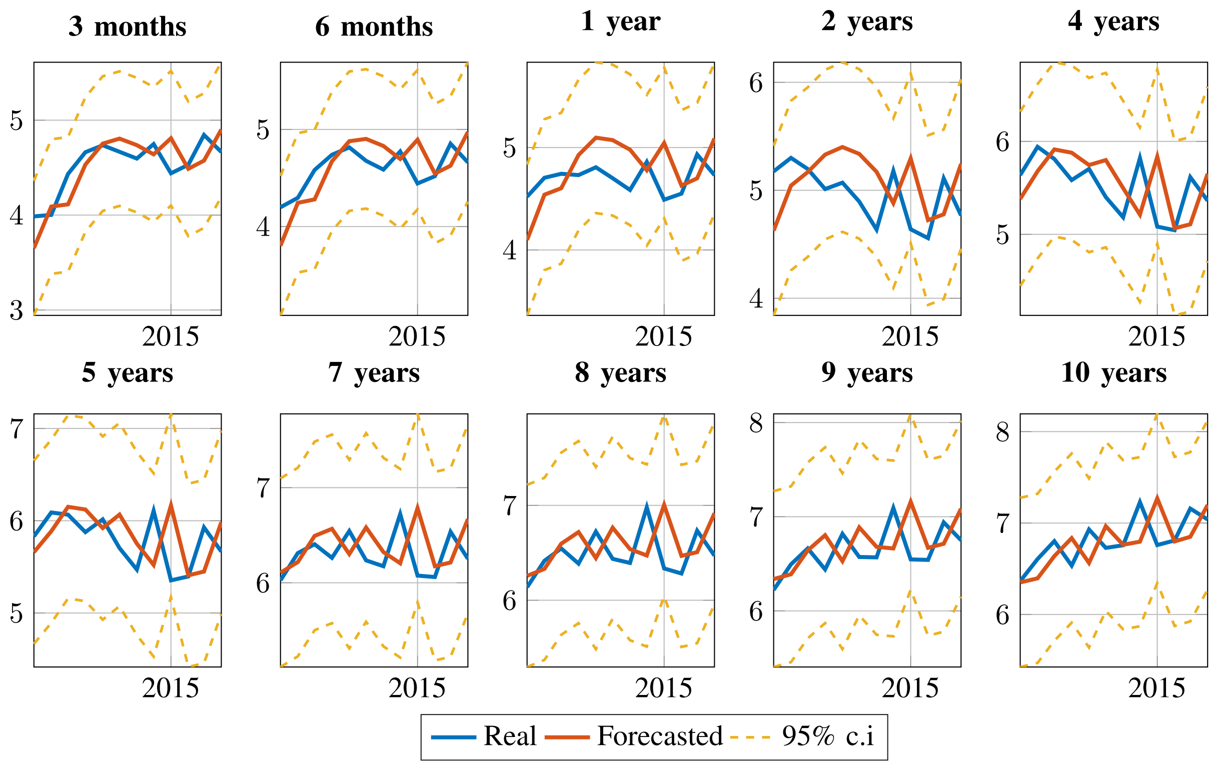

Figure 8. Out of sample one-month ahead forecasts for all maturities with the IMACO-Monetary base model. Yields are reported as continuously compounded with annual periodicity.

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Table I
IN-SAMPLE ERRORS

	Yearly RMSEs (basis points)										
Latent factors model	Average	3m*	6m	1y**	2y	4y	5y*	7y	8y	9y	10y*
	21.31	0.00	24.91	52.25	57.62	23.65	0.00	19.06	19.89	15.71	0.00
Macro-Factors models	Average	3m*	6m	1y**	2y	4y**	5y*	7y	8y**	9y	10y*
TIB-IBR - Monetary base	13.16	0.00	11.83	22.46	27.59	20.14	0.00	17.04	17.77	14.82	0.00
Vix - TIB-IBR	13.20	0.00	12.07	22.39	28.05	20.15	0.00	16.94	17.82	14.63	0.00
IMACO - TIB-IBR	13.23	0.00	11.66	21.94	28.28	20.42	0.00	17.15	17.85	14.97	0.00
WTI - TIB-IBR	13.33	0.00	11.86	22.48	27.53	20.61	0.00	17.77	18.29	14.77	0.00
IPC - TIB-IBR	13.68	0.00	12.00	24.90	29.88	20.20	0.00	17.03	18.02	14.72	0.00
WTI - Monetary base	13.84	0.00	12.07	24.00	29.77	21.28	0.00	18.13	18.17	15.03	0.00
IMACO - Monetary base	14.06	0.00	12.71	25.79	30.75	21.47	0.00	17.15	17.86	14.84	0.00
Vix - Monetary base	14.11	0.00	12.17	23.96	30.49	20.68	0.00	19.27	17.75	16.82	0.00
IMACO - Vix	14.12	0.00	12.94	25.75	31.46	21.51	0.00	17.01	17.94	14.61	0.00
IMACO - WTI	14.23	0.00	12.73	25.80	30.92	21.93	0.00	17.78	18.36	14.76	0.00
USDCOP - CDS 5y	14.30	0.00	12.68	23.67	32.32	20.51	0.00	20.12	17.80	15.89	0.00
CDS 5y - Monetary base	14.35	0.00	12.18	23.94	30.66	20.62	0.00	20.45	17.74	17.89	0.00
IPC - Vix	14.48	0.00	12.53	27.45	34.20	20.94	0.00	17.02	17.98	14.71	0.00
IPC - WTI	14.64	0.00	12.46	27.60	33.84	21.64	0.00	17.63	18.38	14.82	0.00
IMACO - IPC	14.77	0.00	12.98	28.74	34.29	21.65	0.00	17.08	18.12	14.78	0.00
IPC - CDS 5y	14.88	0.00	12.66	27.49	35.68	20.86	0.00	18.79	17.96	15.37	0.00
USDCOP - Vix	15.61	0.00	23.41	25.47	36.04	21.55	0.00	16.80	18.07	14.74	0.00
IMACO - USDCOP	16.07	0.00	15.67	25.18	47.72	21.43	0.00	18.14	17.92	14.63	0.00
CDS 5y - WTI	16.11	0.00	12.31	23.99	33.65	20.48	0.00	34.66	17.59	18.41	0.00
USDCOP - Monetary base	17.42	0.00	17.22	30.11	30.43	21.56	0.00	23.31	27.86	23.75	0.00
IPC - USDCOP	22.04	0.00	12.63	22.80	32.88	20.64	0.00	53.39	17.77	60.27	0.00
Vix - WTI	30.44	0.00	13.40	24.01	107.65	20.51	0.00	80.38	17.68	40.74	0.00
IMACO - CDS 5y	30.78	0.00	17.88	25.73	93.53	21.44	0.00	87.30	17.89	44.07	0.00
CDS 5y - Vix	56.92	0.00	75.79	56.13	151.86	60.20	0.00	70.79	82.35	72.10	0.00
IPC - Monetary base	69.83	0.00	13.38	42.64	108.54	103.98	0.00	178.83	163.53	87.37	0.00
CDS 5y - TIB-IBR	96.95	0.00	75.78	22.33	385.09	20.11	0.00	298.19	17.80	150.24	0.00
USDCOP - TIB-IBR	103.55	0.00	115.69	22.31	546.68	20.15	0.00	237.44	18.30	74.91	0.00
USDCOP - WTI	254.65	0.00	109.17	94.91	169.35	178.33	0.00	925.29	36.50	1032.97	0.00

Models are sorted from best to worst on the basis of their average RMSE across maturities. Yields used in estimation are marked with * (observed without errors) and ** (observed with errors). RMSEs are calculated on annualized yields. Results are calculated using 108 observations (May 2005 to April 2014).

Table II
OUT OF SAMPLE 1-MONTH AHEAD FORECAST ERRORS

	Yearly RMSEs (basis points)										
Latent factors model	Average	3m*	6m	1y**	2y	4y	5y*	7y	8y	9y	10y*
	35.90	24.19	33.13	47.93	51.85	38.94	37.56	34.16	33.42	30.27	27.57
Macro factors model	Average	3m*	6m	1y**	2y	4y**	5y*	7y	8y**	9y	10y*
IMACO - Monetary base	31.83	21.57	24.66	30.87	40.25	38.38	37.53	33.88	33.33	30.18	27.69
IMACO - Vix	31.99	21.72	24.45	30.81	42.36	39.27	37.45	33.28	32.86	30.02	27.68
Vix - Monetary base	32.01	21.35	24.08	30.79	42.60	39.07	37.43	34.07	32.73	30.70	27.31
WTI - Monetary base	32.02	21.57	24.69	31.21	40.37	37.46	37.64	35.62	34.01	30.03	27.61
IMACO - WTI	32.06	21.92	25.25	31.20	39.31	37.58	37.62	35.13	34.27	30.72	27.64
USDCOP - Monetary base	32.19	27.32	29.03	26.66	37.01	39.28	37.39	34.60	33.42	29.50	27.70
CDS 5y - Monetary base	32.39	21.24	23.74	30.38	42.88	39.51	37.40	35.28	34.18	32.08	27.18
IMACO - IPC	32.92	22.11	26.07	34.56	44.00	38.85	37.54	34.31	33.64	30.44	27.65
CDS 5y - WTI	32.99	21.32	24.74	30.87	39.18	40.38	37.26	42.92	34.24	31.15	27.78
IPC - WTI	32.99	22.15	26.15	34.60	43.20	38.12	37.62	35.36	34.37	30.73	27.63
IPC - Vix	33.10	21.88	25.54	34.87	46.85	39.71	37.47	33.91	33.12	30.03	27.63
USDCOP - CDS 5y	33.24	21.56	23.29	29.40	49.07	39.52	37.55	40.56	34.07	29.78	27.59
Vix - TIB-IBR	33.27	20.64	23.89	35.93	51.16	39.74	37.47	33.37	32.91	29.99	27.62
TIB-IBR - Monetary base	33.38	20.91	25.17	36.84	48.95	39.76	37.50	33.56	33.24	30.18	27.68
WTI - TIB-IBR	33.64	21.13	25.48	37.22	48.72	38.69	37.60	35.07	34.28	30.62	27.63
IMACO - TIB-IBR	33.65	20.84	24.66	35.75	49.34	40.71	37.47	33.28	34.04	32.46	27.91
IPC - CDS 5y	33.83	21.62	24.74	34.45	50.22	39.92	37.53	37.66	34.39	30.24	27.54
IPC - TIB-IBR	34.22	21.42	25.95	39.14	52.72	39.88	37.53	34.20	33.48	30.26	27.63
IMACO - USDCOP	38.25	27.94	31.21	30.20	88.29	38.19	37.66	37.56	33.37	30.36	27.71
IPC - USDCOP	39.77	20.86	22.52	28.87	47.24	37.64	37.53	69.88	32.74	71.68	28.79
USDCOP - Vix	49.73	132.66	79.68	33.44	53.11	39.77	37.00	31.92	32.03	29.63	28.03
IMACO - CDS 5y	51.85	28.83	32.89	32.89	128.83	39.17	38.60	107.95	32.81	48.88	27.66
Vix - WTI	52.72	23.13	27.08	33.71	153.00	38.81	38.12	93.25	36.68	56.09	27.33
IPC - Monetary base	89.10	23.65	29.15	44.50	107.52	105.99	40.34	212.52	193.59	104.59	29.15
CDS 5y - Vix	97.35	203.82	134.97	56.53	155.30	64.18	37.71	92.63	106.04	92.41	29.92
CDS 5y - TIB-IBR	107.24	70.12	84.84	50.16	363.89	38.83	42.02	241.81	34.26	117.18	29.26
USDCOP - TIB-IBR	128.52	117.91	147.30	60.37	543.93	39.69	42.52	209.10	31.95	64.86	27.54
USDCOP - WTI	326.06	122.04	178.20	94.54	188.98	208.75	63.23	956.57	146.98	1100.36	200.96

Models are sorted from best to worst on the basis of their average RMSE across maturities. Yields used in estimation are marked with * (observed without errors) and ** (observed with errors). RMSEs are calculated on annualized yields. Results are calculated using 12 observations (May 2014 to April 2015).