Sports betting odds: A source for empirical Bayes

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Abstract

This paper proposes a new betting strategy using three different variations of the Kelly criterion, it is shown an elicitation procedure based on betting odds to find the hyperparameters of the prior distributions that are used to predict the outcomes of the premier league for the two different seasons using the Categorical-Dirichlet with the addition of historical information. Our results shows that betting according to our elicitation gives a better performance compared with others proposed methodologies in the literature, specifically we show a profit of 98% betting in a total of 380 matches in a two year period.

Keywords: Betting Odds, Elicitation, Simulation, Betting Strategy, Premier League.

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1 Introduction

Today, in a world where there are loads of fans of every kind of sports and also a growing number of bets online, there is an important issue in the area of sport forecasting [Stekler et al., 2010]. In fact, there are different kinds of sources for forecasting such as predictions from the experts, prediction markets and betting bookmarkers.

In the literature there is a lot of works regarding to sports forecasting, but different approaches are used for different kinds of sports. For instance, there is a model for basketball using a Markov model [Štrumbelj and Vračar, 2012] and finally there are different approaches for football such as one based on betting odds [Strumbelj and Sikonja, 2010].

Sports forecasting have had a lot of attention, there are different ways for approaching this matter, first, we mention the statistical models which uses sport-related inputs, then the expert tipsters [Štrumbelj, 2014], also there are the ones which are based on prediction markets [Spann and Skiera, 2009] and finally the models that are based on betting odds [Vlastakis et al., 2009].

One can say that football is the king among the sports. In fact, there is no doubt about its popularity in the world, and behind that there is a big betting market which moves a few billion dollars, and that phenomenon has happened since its introduction online [Constantinou et al., 2013], so there is no surprise in the special attention and the study on this market, with two different points of view, from an academic scope and from individuals which are looking for profits in an inefficient market.

Bets have been used for different issues and there is also empirical evidence suggesting that they are the best (most accurate) public source of information for sports forecasting [Štrumbelj, 2014]. On their work they also conclude that forecasting from betting odds are better than any other approach, or at least as good.

Spann and Skiera [2009] conclude on their work that betting odds are the best source for sport forecasting but we can see at Štrumbelj [2014] that there are different ways to obtain the associated probability from a betting odd and they conclude that the Shin [1993] methodology is the most accurate for calculating the probabilities among the different methods that have been used before.

Peter F. Pope [1989] worked with recommendations from some experts, 25 years ago when there was not the availability of the online market, and he showed that those experts opinions were more accurate by the end of the season. Rue and Salvesen [2000] used a Bayesian dynamic generalized linear model against the odds of one particular bookmarker and showed a good profit in the case of the BPL. On Forrest et al. [2005] there is evidence regarding the outperform of the statistical models against expert predictions. They also concluded that the by the end of the season the odds setters learned from the season and had a better performance.

In this paper the methodology is showed in Section 2 explaining what we used. First of all, there is the Bayesian model which is the root of this work, then we explain an strategy based on the Kelly criterion, next we explain the main novel which is determining λ in what we consider the "λ-Kelly" criterion. Section 2 shows the results of how the betting strategy would have performed in the last two seasons, and a comparison between the Kelly criterion using different strategies including our proposal using the Bayesian model results. At the end we conclude on Section 4 if the strategy is profitable or not, and if the "λ-Kelly" criterion its more trustworthy than the complete Kelly.

2 Methodology

Our Bayesian approach to predict the possible outcomes is the Categorical-Dirichlet model. In particular, we assume that the likelihood is given by a categorical distribution with three possible outcomes $\{Win, Draw, Loss\}$, that is, $p(x = i) = p_i$, $x = \{Win, Draw, Loss\}$, $\sum_i p_i = 1$. On the other hand, prior information is summarized in a Dirichlet distribution such that $\pi(\boldsymbol{p}) \sim \mathcal{D}(\boldsymbol{\alpha})$, that is, $\pi(\boldsymbol{p}) = \frac{\Gamma(\alpha_1+\alpha_2+\alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)} p_1^{\alpha_1-1} p_2^{\alpha_2-1} p_3^{\alpha_3-1}$.

Following Bayes's theorem, the posterior distribution is $\pi(\boldsymbol{p}|Data) \sim \mathcal{D}(\boldsymbol{\alpha} + \boldsymbol{c})$ where $\boldsymbol{c} = (c_1, c_2, c_3)$ is the vector with the number of occurrences of category i, $c_i = \sum_{j=1}^{N} [x_j = i]$.

$$\pi(\boldsymbol{p}|Data) = \frac{\Gamma(\alpha_1 + \alpha_2 + \alpha_3 + N)}{\Gamma(\alpha_1 + c_1)\Gamma(\alpha_2 + c_2)\Gamma(\alpha_3 + c_3)} p_1^{\alpha_1+c_1-1} p_2^{\alpha_2+c_2-1} p_3^{\alpha_3+c_3-1} \tag{1}$$

We use betting odds to obtain the hyperparameters of our prior distributions using Dirichlet models [Thomas and Jacob, 2006; Hizaji and Jernigan, 2009]. In particular, we take the betting odds associated with the games from different online bookmakers, and transform them into outcome probabilities using the procedure developed by Shin [1991, 1993] but using the procedure by Jullien and Salanié [1994]. The reason for this decision is the well known argument that betting odds are inherently biased due to bookmakers' profits from their service, thus they offer unfair odds. More specifically, the booksum (the sum of the inverse odds) is greater than 100%, and as a consequence, the quoted odds overestimate the probability of every possible outcome [Strumbelj, 2014]. A common practice is to use a procedure called "basic normalization"; however, Strumbelj [2014] shows that the probabilities deduced from betting odds using the Shin [1993] procedure are more accurate than the probabilities produced by basic normalization.

In particular, Shin [1993] develops a model where bookmakers quote odds such that they maximize their expected profit. Therefore, following the methodology proposed by Shin [1993], the probability estimates from the betting odds for a specific game $O = \{o_{Win}, o_{Draw}, o_{Loss}\}$ are given by

$$p(z)_i^{betting\ odds} = \frac{\sqrt{z^2 + 4(1-z)\frac{(1/o_l)^2}{\sum_l (1/o_l)}} - z}{2(1-z)},\ i = \{Win, Draw, Loss\}, \tag{2}$$

so that $\sum_{i=1}^{3} p_i^{betting\ odds} = 1$.

Following Jullien and Salanié [1994], we find the profit margin z,

$$Arg\min_z \left\{ \sum_{i=1}^{3} p(z)_i^{betting\ odds} - 1 \right\}^2. \tag{3}$$

In addition, we use historical information about every contender to build the categorical likelihood for each game. Thus, we count, for each pair of contenders, their numbers of wins and losses (and draws if possible) when they met in previous matches.[1] Finally, we got the posterior Dirichlet distribution for each game, and used it to simulate the possible outcomes from a categorical distribution whose outcomes are *Win*, *Draw* (when it applies), and *Loss* for each game.

From (1) there is our methodology to estimate the probability but we have not proposed anything about the betting strategies. For that duty we propose to use the Kelly criterion.

[1] We downloaded this information from [http://www.fifa.com/, http://www.soccerbase.com/ and http://tennis.wettpoint.com/].

Kelly [1956] based his formulation in the following equation:

$$f^* = \frac{\theta p - 1}{\theta - 1} \tag{4}$$

where θ is the given odd, p is the probability of the outcome and f^* is the fraction of our budget that we are supposed to bet according to this criterion.
Note that there is no restriction for f^* so there when $p < \frac{1}{\theta}$ f^* becomes a negative fraction, but there is no possibility for betting a negative amount of money, so it shows that we only bet when $p \geq \frac{1}{\theta}$

Kelly criterion has some great asymptotic properties but it could lead to a big loss in a few bets in a bad streak, so that is the reason for some people to use bet a fraction lesser than f^*. The most common choice is to bet only the half of the Kelly fraction which is known as the "half Kelly". Using the latter we win security but we will expect more bets for obtain a good profit. It is not recommendable to use a bigger fraction because over-betting is much more penalized than under-betting with this methodology [Thorp, 1998].

Kelly criterion has some problems in order to be used in a real context. The biggest issue is that fact that the probability is not known for most events where you can bet, and also, if it is possible to know the probability, the book-marker, who is the one who defines the odd (θ), will offer an odd that is profitable for their business not for the gambler. So the idea is that after our estimation process it would be possible to have a good approximation for the probability p and also we can use to fix the problem that we have showed before.

As we said before there are some variations of the Kelly method such as the "half" Kelly but, why half? We consider the following equation:

$$\lambda f^* = \frac{\theta p - 1}{\theta - 1} \tag{5}$$

where $\lambda \in [0, 1]$.

From (5), if we consider $= 1$ or $\lambda = 0.5$, we obtain the Kelly or the half Kelly respectively. To determine which λ is the more profitable we will use three different approaches:

1. Stopping loss in a matchday

2. Fixed expected win with simulation each matchday

3. Maximize the expected win with simulation each matchday

The first idea is to consider the worst scenario which is when we lose the bet in all the games in the matchday, so if we are using the Kelly criterion we will end with a fraction of our initial bankroll (the beginning of the matchday, not the beginning of the bets) given by the following equation:

$$\alpha = 1 - \prod_{i=1}^{m}(1 - f_i^*) \tag{6}$$

where f_i^* is the Kelly fraction for the game i of the matchday, α is the total loss in the worst scenario and m is the number of games in each matchday.
The idea is to use (5) and (6) to choose a λ which secures a maximum allowed loss (α^*), so, the idea is that we choose a λ which leads to:

$$\alpha^* = 1 - \prod_{i=1}^{10}(1 - \lambda f_i^*) \tag{7}$$

and we consider

$$Arg \min_{\lambda} \left\{ -\alpha^* + 1 - \prod_{i=1}^{10}(1 - \lambda f_i^*) \right\}^2 . \tag{8}$$

From the optimization process in (8) we obtain the λ which satisfies (7). So the idea is that given an α^* it could be used to define a different λ for each matchday.

The second idea to choose λ is really simple, the idea is to choose an expected win β, then we simulate the matchdays from a multinomial distribution using the estimated probabilities and also calculate the profit for that scenario, then we repeat that 10.000 times so we obtain the expected win (w). The idea is to choose the minimum λ which satisfies $w \geq \beta$.

The third idea is to proceed as in the second one, but here we consider λ to maximize w. Both, the second and the third one are done following the pseudocodes in Algorithms 1 and 2 in the appendix .

3 Results

After gathering the data and the implementation of the model we have some results as we see in Table 1, there are some important changes between some probabilities calculated with the Shin process and our posterior, in fact, those differences goes up to 0.07 in the 14-15 season.

	Shin			Posterior		
Match	Home	Draw	Away	Home	Draw	Away
Arsenal Vs Crystal Palace	0.7740	0.1562	0.0696	0.7733	0.1559	0.0708
Leicester Vs Everton	0.3076	0.2871	0.4052	0.3122	0.2915	0.3963
Man United Vs Swansea	0.7166	0.1906	0.0927	0.7142	0.2009	0.0849
QPR Vs Hull	0.3877	0.2972	0.3150	0.3786	0.3086	0.3128
Stoke Vs Aston Villa	0.4941	0.2838	0.2219	0.4607	0.2979	0.2415
West Bromwich Vs Sunderland	0.4272	0.2904	0.2822	0.4089	0.2936	0.2975
WestHam Vs Tottenham	0.2584	0.2769	0.4646	0.2630	0.2608	0.4762
Liverpool Vs Soton	0.7195	0.1846	0.0958	0.7013	0.1906	0.1081
Newcastle Vs Man City	0.1802	0.2348	0.5848	0.1921	0.2460	0.5619

Table 1: Estimated probabilities for the first matchday PL14-15.

The differences between Shin and the posterior results are caused by the addition of the extra information (historical results). The differences between the results are bigger if the historical results were really different in the past. For example the history said that in the past a match between Arsenal and Aston Villa might have been quite tough, but the odds said that back in February 2^{nd} it was going to be an easy victory for Arsenal. We can see in Figure [2] that if we consider the historical information there will be some big differences in the prior, and so in the posterior.

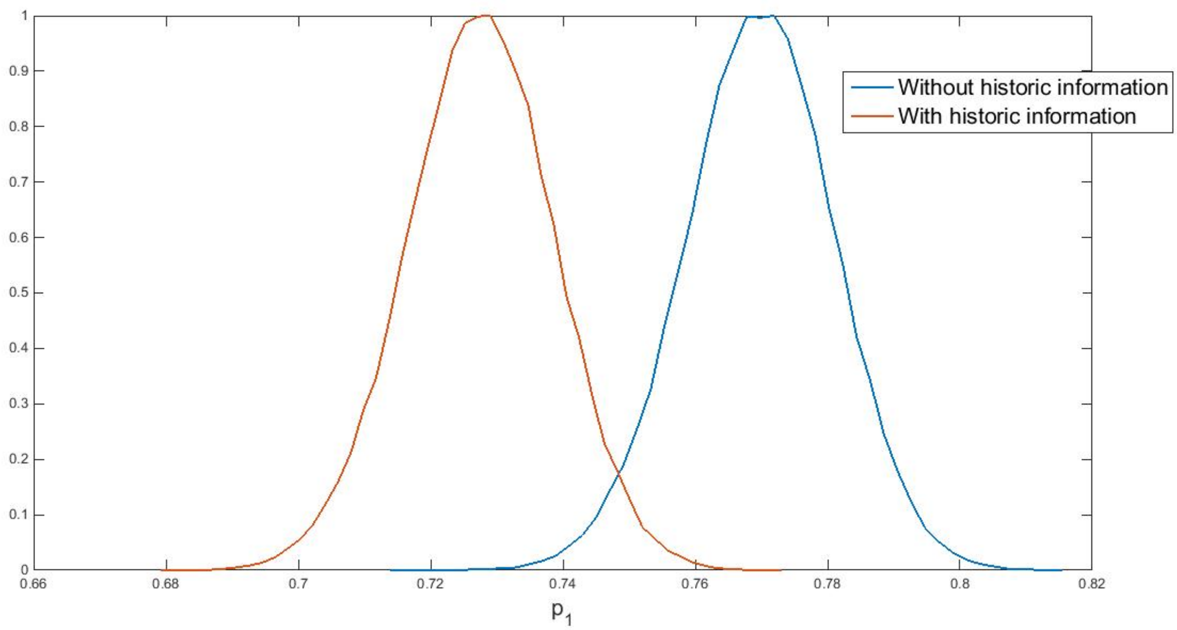

Figure 1: Differences between prior and posterior.

In the literature the estimations that had been made with the Shin procedure have had good results so it is a good idea to use the mean squared error (MSE) for both[de Finetti, 1972; Suzuki et al., 2010; Soares and Correa, 2013], the Shin process and our elicitation process, and see if there are significant differences.

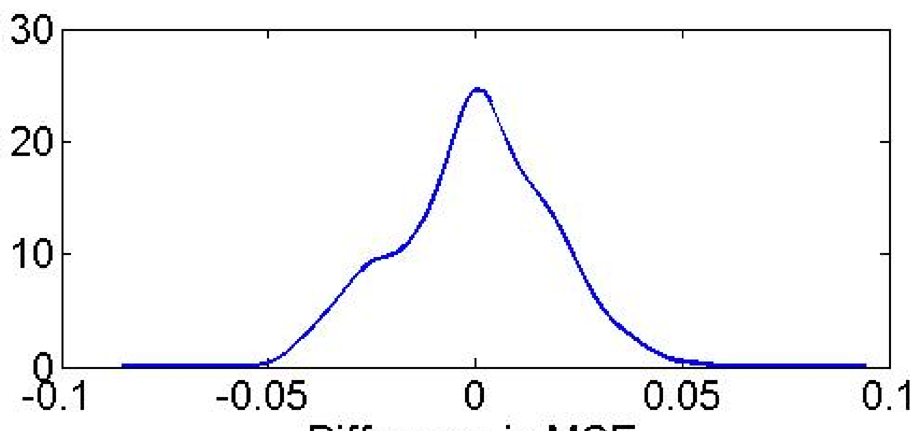

Figure 2: Comparison against Shin's process.

Where the mean difference is $-2.2x10^{-5}$ and the standard deviation is 0.018, so even when this sample give us a better MSE measure, there are not significant differences between both estimations in terms of forecast.

We simulated the process of betting following the Kelly criterion, those results are shown in Figure [3]:

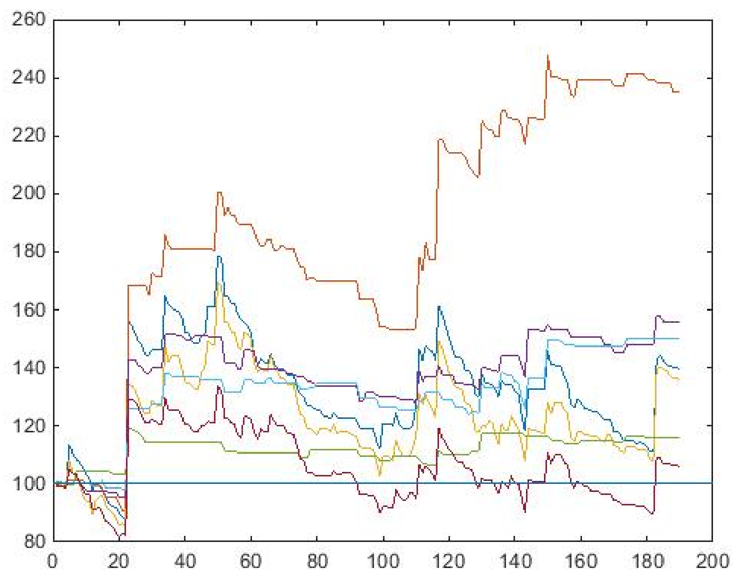

Figure 3: Process of betting with Kelly criterion in the 14-15 season.

As we can see in Figure [3] the results are different in each bookmarker, but in general our strategy is profitable in almost every period of time in each bookmarker, we only have a poor perfomance in Bet365 but only in some specific periods of time.

We also simulate the case in which we bet always the same amount of money (1 US) when $p_i > \frac{1}{\theta_i}$ (we consider this as the simplest ans most intuitive case) and we see the results in Figure [4].

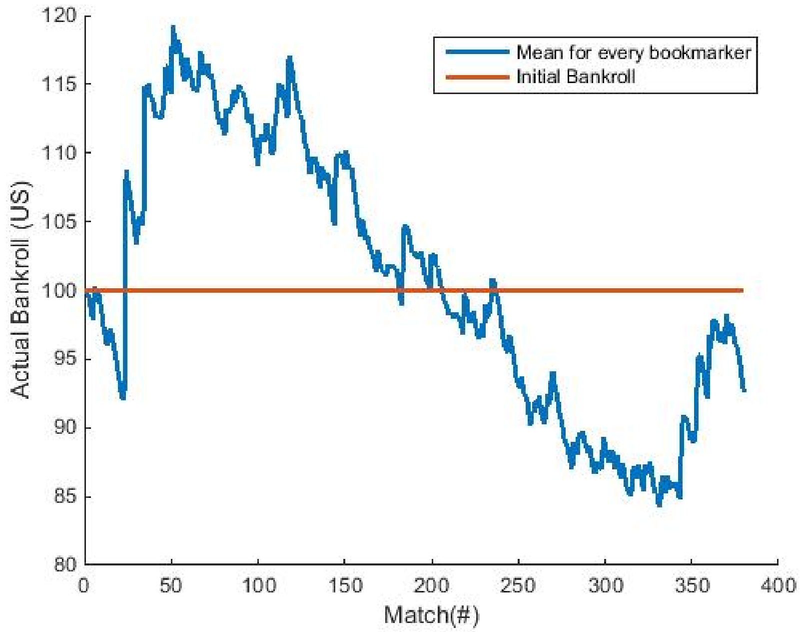

Figure 4: Betting for the greatest $p_i\theta_i$ each match PL 14-15 .

We can see in Figure [4] that just in one season this simple method give us different streaks so, is not exactly a method which everyone can trust, it could give you good results but you can result in the loss of money too.

We can see in Figures [5] and [6] that there is evidence for the case when betting one dollar in the 14-15 season, betting always for the biggest odd where profitable, but there is not evidence that tendency works for every season, in fact, we can see in Figures [7] and [8] the opposite case, betting for the biggest odd were not profitable but betting for the most likely were, so we can see that these strategies, which are really simple, are not exactly robust.

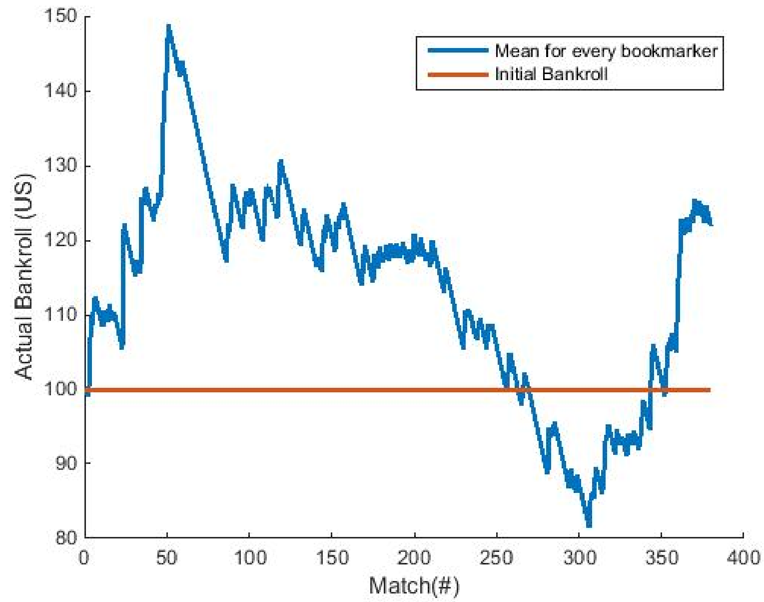

Figure 5: Mean when betting for the biggest odd every game PL 14-15.

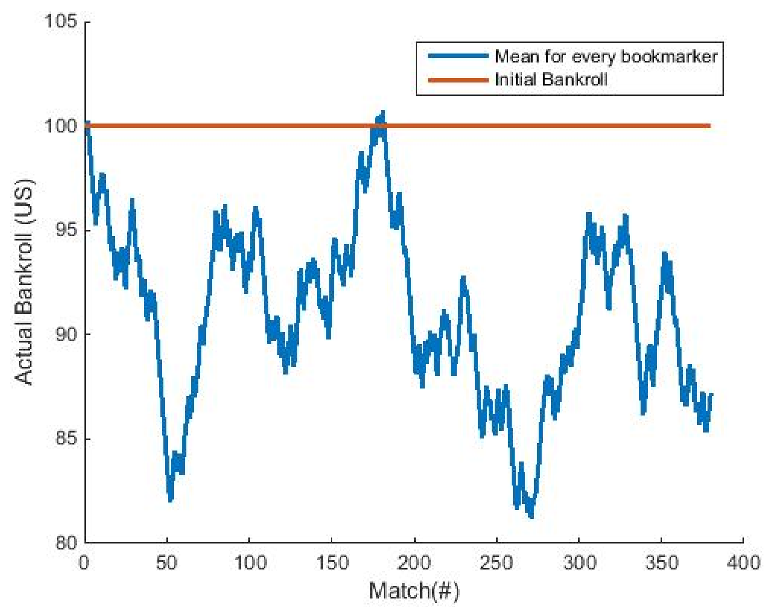

Figure 6: Mean when betting for the better probability every game PL 14-15.

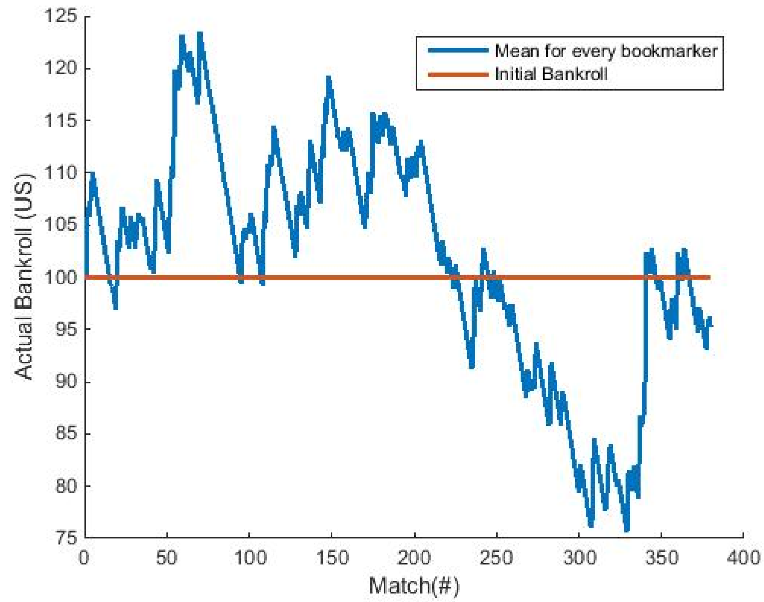

Figure 7: Mean when betting for the biggest odd every game PL 13-14.

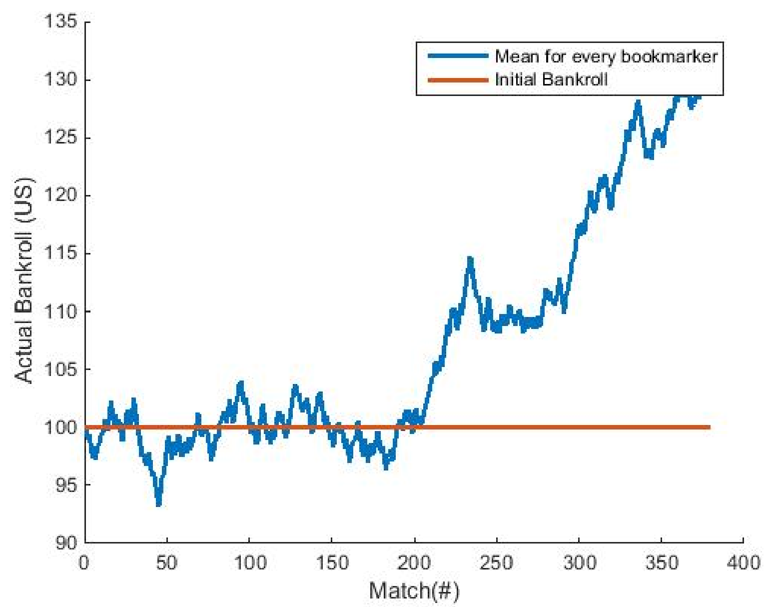

Figure 8: Mean when betting for the better probability every game PL 13-14.

So far we have only showed that using our estimation process and the Kelly criterion is a profitable betting strategy, but the question here is: How good is it? We also consider using the

Shin and the normalization process with the Kelly criterion.

Specially the Shin process had been proved to be a good estimator of the sports probabilities, but we can see in Figure [10] and [9] that for this case there is no big differences between the Shin process and a normalization process but what we want to emphasize here is that our process had a much better performance in terms of profitability, but is meaningful to notice that by the end of the 13-14 season our process is humbled by the normalization process but betting with our estimation would had produce a bigger profit in almost point of time.[2]

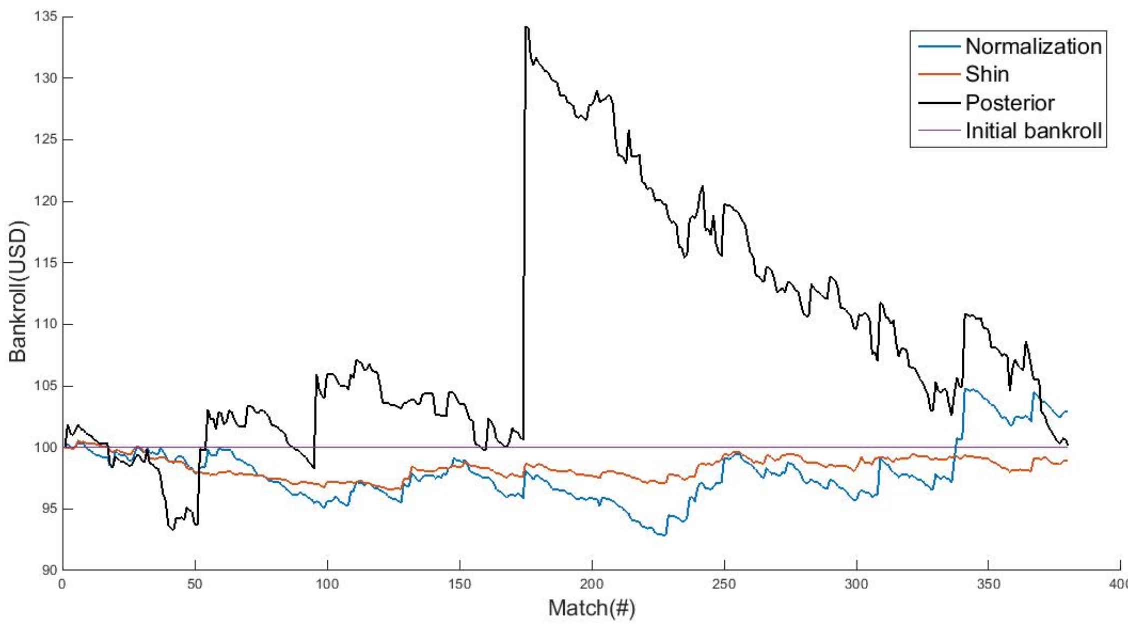

Figure 9: Testing our process in the 13-14 PL season.

[2]For now on, the results are the mean in 8 different bookmarkers

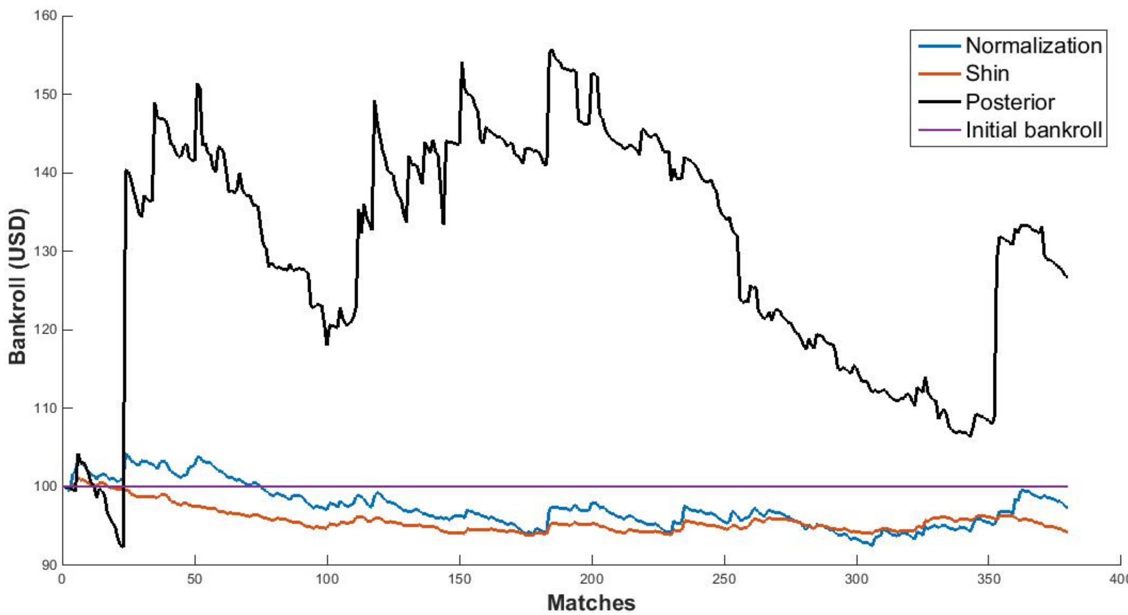

Figure 10: Testing our process in the 14-15 PL season.

For both, the 13-14 and 14-15 season the results showed that they would had been growing in a good rate until the half season, as we said before, [Forrest et al., 2005; Peter F. Pope, 1989] conclude that as the season goes the bookmarker's predictions (or the expert's) improves, so it is really plausible that as the season goes it is more difficult to win against market. On Figure [11] is shown how the betting strategy would had perform only in 190 matches on each season.

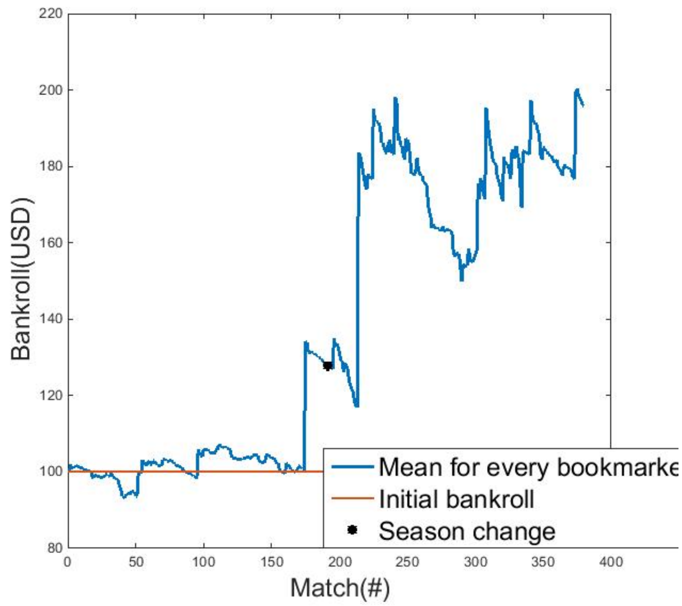

Figure 11: betting until the half of both seasons.

Until this point every gambling strategy had been based on the Kelly criterion, particulary, the full Kelly, but we have proposed three different ways to perform with a λ-Kelly criterion, in Figures [12],[13],[17],[14],[15] and [16] how would those strategies had perform.

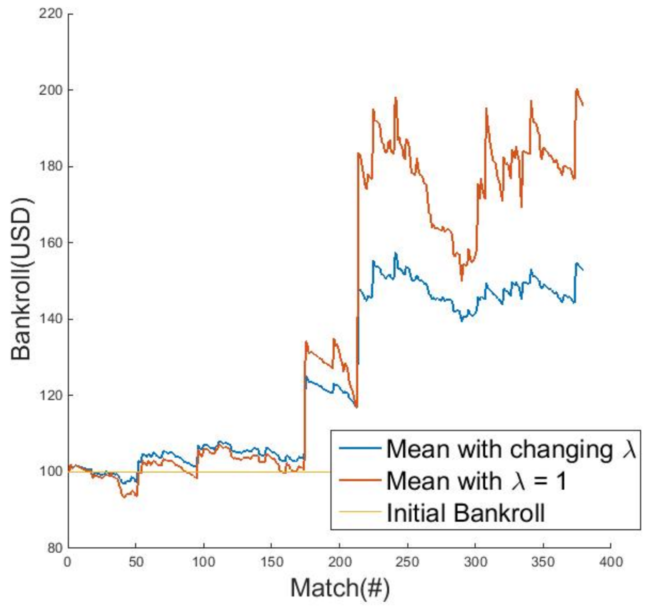

Figure 12: Stopping loss 5%.

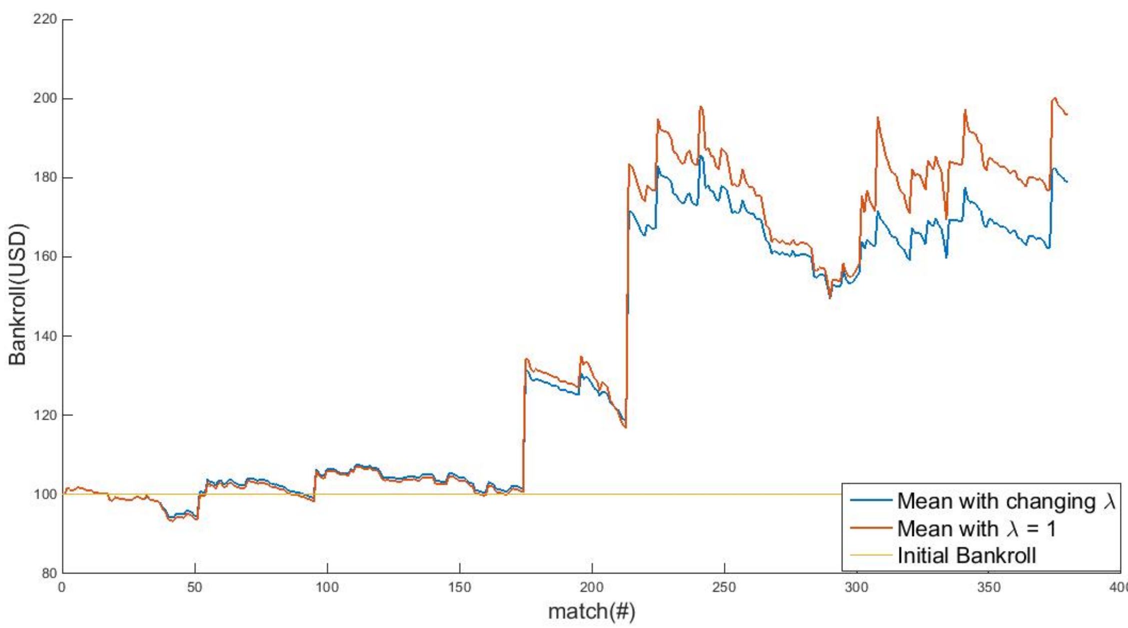

Figure 13: Stopping loss 10%..

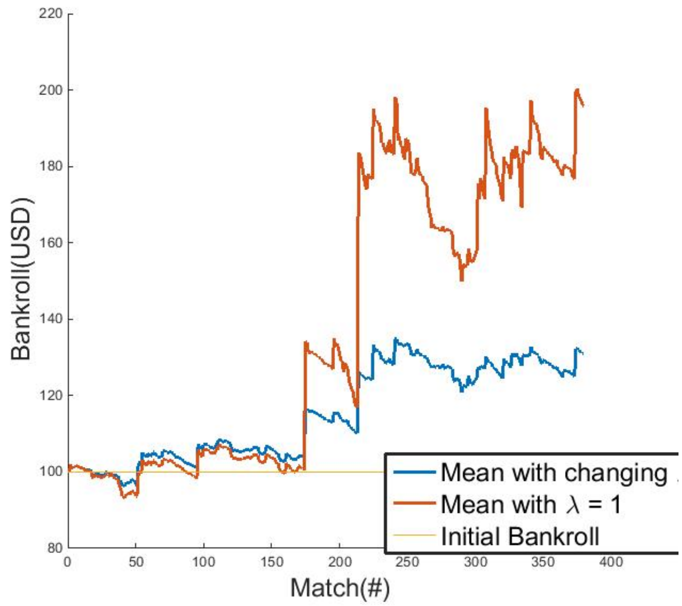

Figure 14: Expecting profit about 20%.

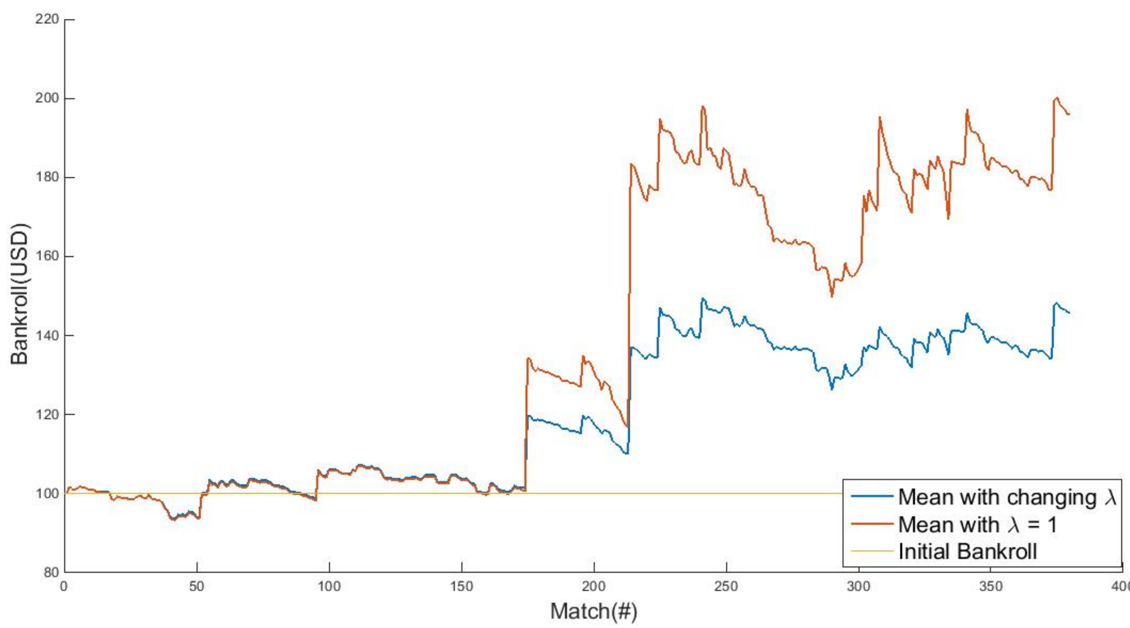

Figure 15: Expecting profit about 40%.

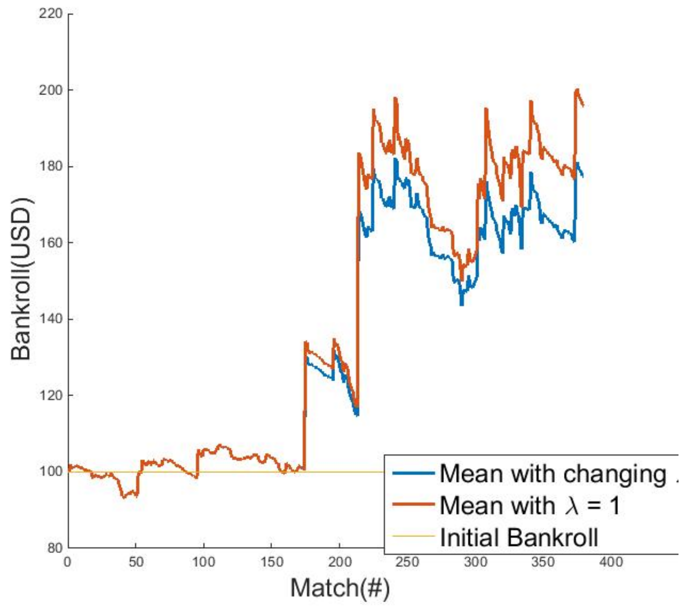

Figure 16: Expecting to double initial bankroll.

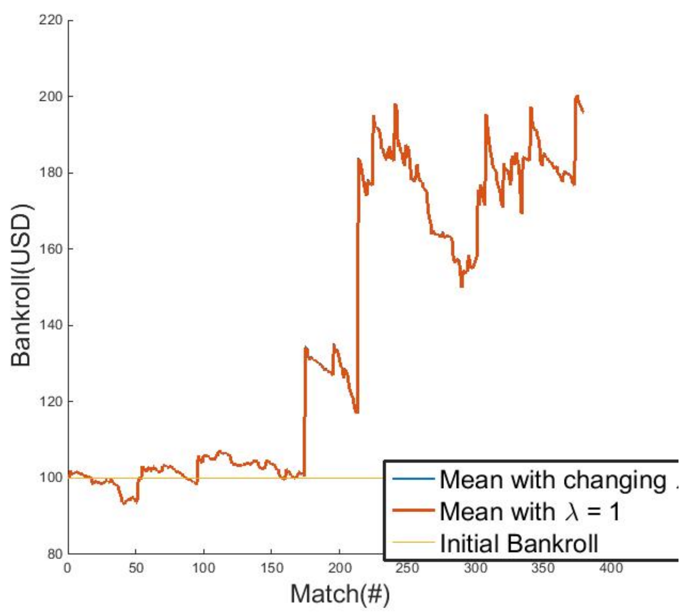

Figure 17: Maximise expected win.

Using a λ-Kelly strategy change only the amount of money that the criterion suggest to bet, not when to bet. So that is the reason of why there are no differences in when do the strategy wins or losses, the difference is only in the amount of money in each case. On Figure [12] and [14] there is evidence that using a more cautious strategy gives a better performance when there is a bad strike, but, it leads not to have biggest winnings when there are big opportunities according to the Kelly criterion.[3] From Figure [16] we can conclude that according to the estimated probabilities we are expecting to have bigger profits than 100% but, as we can see, there is no evidence about that kind of performance, so it is easy to conclude that there is a bias on our elicitation. On Figure [17] there is no differences between the full Kelly and the proposed λ-Kelly, in fact, every λ with this process were either 1 or 0 (being 0 only on 1 match-day), shows that when there is an expected win with this process, the best expected results is when using the full Kelly or not to bet in an entire match-day in some cases.

4 Concluding Remarks

First of all it is really important to notice that there is no significance loss in term of prediction power when comparing our process with the Shin one, once we said that, the simulations showed that simple gambling strategies leads not to have good profits, also it is really important the fact that there when using strategies that only suggest when to bet does not gives good returns, so that is one reason of why the Kelly criterion is a good approach of how much to bet, and as we can see in this paper, it leads to a good profit.

The results showed that how was noticed by Forrest et al. [2005] and Peter F. Pope [1989] by end of the season the experts have better performance on their prediction which leads our betting strategy not to be profitable by the end of the season, so that is one reason not to bet on that part of the season.

Our proposals for a λ-Kelly strategy gave different profit rates according the risk aversion, but in general the three of our proposals give good returns and some have a better performance when there is a bad strike, so that could be a reason to someone who is very averse to risk not to choose the full Kelly strategy, but the results showed that when using the full Kelly the person

[3] When $p_i\theta_i >> 1$.

is maximise the expected return which is what almost every gambler rather.

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Appendix

Algorithm 1 simulates an entire matchday given the estimated probabilities using a multinomial distribution.

Algorithm 1 pseudocode escenario simulation

```
Read
N=length(P)
for i = 1 to N do
    Escenario(i)=multinomial(P(i))
end for
return Escenario
```

Algorithm 2 simulates how would the profit for a matchday would have been given a random scenario.

Algorithm 2 pseudocode expected profits

```
Read λ , iter, Kelly, Bet, P,Odds
Profit0=100
for i = 1 to iter do
    Profit=Profit0
    Escenario=Escenario simulation (P)
    N=length(Escenario)
    for j=1 to N do
        Profit=Profit*(1-Kelly(j))
        if Bet(j)=Escenario(j) then
            Profit=Profit+Profit*Kelly(j)*Odds(j,Bet(j))
        end if
    end for
    Profits(i)=(profit-profit0)/Profit0
end for
return mean (Profits)
```