Vessel Extraction Using the Buckmaster-Airy Filter

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November 24, 2015

Abstract

A new technique for vessel extraction from biomedical images using the so called Buckmaster-Airy Filter is designed, prototyped and tested. The design, the prototyping and the testing were performed using computer algebra software, specifically the Maple package *ImageTools*. Some preliminary experiments were performed and the results were very good. Our new technique is based on partial differential equations. Specifically two dimensional Airy diffusion equation and the two dimensional Buckmaster equation were used for designing the new Buckmaster-Airy Filter. Such new filter is able to enhance the quality of an image, producing simultaneously noise elimination, but without altering the edges of the image. The new Bukmaster-Airy filter is applied to the target image via discrete convolution. The results of some experiments of vessel extraction will be presented.

Keywords: Vessel extraction, angiogram, rizoma structure, Buckmaster equation, Airy diffusion, skeletonization, Buckmaster-Airy filter, computer algebra, ImageJ.

1 Introduction

In a recent work the so called Airy-Kaup-Kupershmidt filter was applied to digital image processing[1]. The Airy-Kaup-Kupershmidt filter was designed applying the Kaup-Kupershmidt operator [2] on the solution of the Airy diffusion equation [3, 4]. In [1] was showed that the Airy-Kaup–Kupershmidt filter is a powerful edge detector [5, 6] and is also a powerful enhancement tool in image processing.

Also in a recent work the so called Korteweg-de Vries-Kuramoto-Sivashinsky filter was applied to biomedical image processing [7]. The Korteweg-de Vries-Kuramoto-Sivashinsky filter was designed applying the Kuramoto-Sivashinsky operator [8] on the Korteweg-de Vries soliton [9]. In [7] was showed that the

Kuramoto–Sivashinsky-Korteweg–de Vries Filter is a very good tool for enhancement of images.

From other side, other classes of new filters have been introduced recently in digital image processing: Bessel filters were proposed in [10], Morse filters were applied in [11], Ornstein-Uhlenbeck filters were studied in [12] and some quantum filters were presented in [13, 14, 15].

All these new filters are inscribed in the field of applications of partial differential equations in digital image processing [16]. Many partial differential equations extracted from fluid dynamics, heat transfer, mass transfer and quantum mechanics can be applied in digital image processing.

In the present work, we are continuing such trend of research. Specifically, we will use the Buckmaster equation [17] of the fluid dynamics in combination with the Airy propagator. Our objective is then to present a new filter, which will be named the Buckmaster-Airy filter, and its possible applications as a new technique for vessel extraction from biomedical images.

Vessel segmentation and extraction are important tools in the analysis of biomedical images of the circulatory blood vessels [18] and other dendritic or rizoma biostructures. Vessel segmentation is not an easy task because it depends of vessels width, image resolution, contrast, brightness and other image features. Currently there are different types of methods to perform the vessel segmentation but there is not a method that works with every medical image modality. Using partial differential equations and special functions of mathematical physics, we will design a new filter which can be used to improve visualization of blood vessels for black and white images.

In the present work, the Buckmaster-Airy filter will be prototyped using computer algebra software and some experiments of vessel extraction will be performed using the Maple package ImageTools [19] and the program ImageJ [20].

2 Problem

In this work, we consider an important problem in digital biomedical image processing. Such problem is concerned with the extraction of vessels and other rizoma structures from biomedical images such as angiograms and rizomagrams. Vessel segmentation and extraction are important tools in the analysis of biomedical images of the circulatory blood vessels which are key components to automated radiological diagnostic system and in general for diagnosis of vascular disease, such as malformations.

By extraction of vessels we understand the clear extraction of the rizoma structure of the vessel in such way that the vessel is clearly delimited in its filaments. Given that the vessel is a rizoma object is necessary to obtain at first instance an appropriate enhancement of its filaments which is reached using appropriate filters capable to provide at the same time smoothing and enhancement. Such combination of smoothing and enhancement must be obtained using determined operators which are applied to the images via some kind of diffusive and coherent derivative. Our problem consists in finding an appropriate diffusive and coherent derivative which let us to obtain excellent vessel extraction using the combination of rizoma detectors with skeletonization. The method used to solve the formulated problem will be presented at the next section.

3 Method

We will use partial differential equations in biomedical digital image processing. Partial differential equations have several applications, especially in models with diffusion or wave motion. Specifically, we will use the two dimensional Airy diffusion equation, which serves to describe linear dispersion in terms of a third-order partial differential equation; and the two dimensional Buckmaster equation which describes thin viscous fluid sheet flow.

The one-dimensional Airy equation has the form

$$\frac{\partial}{\partial t}P(x,t) = \eta\left(\frac{\partial^3}{\partial x^3}P(x,t)\right) \tag{1}$$

with the initial condition

$$P(x,0) = \delta\left(x - X\right) \tag{2}$$

where δ is the Dirac delta function.

The problem (1 - 2) is solved using the Fourier transform $\mathscr{F}$. Taking the Fourier transform of (1) respect to x we have that

$$\frac{\partial}{\partial t}\mathscr{F}\{P(x,t),x\} = -i\eta k^3\mathscr{F}\{P(x,t),x\} \tag{3}$$

where

$$\mathscr{F}\{P(x,t),x\} = \frac{\sqrt{2}}{2\sqrt{\pi}}\int_{-\infty}^{\infty} P(x,t)e^{-ikx}\,dx \tag{4}$$

In the Fourier domain, the initial condition (2) takes the form

$$\mathscr{F}\{P(x,0),x\} = e^{-ikx} \tag{5}$$

Making the substitution $\mathscr{F}\{P(x,t),x\} = P(t)$, Equation 3 is rewritten as

$$\frac{\partial}{\partial t}P(t) = -i\eta k^3 P(t) \tag{6}$$

and (5) adopts the form

$$P(0) = e^{-ikx} \tag{7}$$

Then, solving (6) with the initial condition (7) we obtain

$$P(t) = e^{-iXk}e^{-I\eta k^3 t} \tag{8}$$

The inverse Fourier transform of Equation 8 is computed as

$$P(x,t) = \frac{\sqrt{2}}{2\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-iXk}e^{-i\eta k^3 t}e^{-ikx}\,dk \tag{9}$$

and the result is

$$P(x,t) = \frac{\sqrt{-\frac{x-X}{(\eta t)^{1/3}}}\,K_{1/3}\left(\frac{2\sqrt{3}\left(-\frac{x-X}{(\eta t)^{1/3}}\right)^{3/2}}{9}\right)}{3(\eta t)^{1/3}\pi} \tag{10}$$

where K is the Bessel K function. The solution (10) can be rewritten as

$$P(x,t) = \frac{3^{2/3} Ai\left(\frac{3^{2/3}(-x+X)}{3\eta^{1/3}t^{1/3}}\right)}{3\eta^{1/3}t^{1/3}} \tag{11}$$

Now, the two-dimensional Airy equation has the form

$$\frac{\partial}{\partial t}P(x,y,t) = \eta_1\left(\frac{\partial^3}{\partial x^3}P(x,y,t)\right) + \eta_2\left(\frac{\partial^3}{\partial y^3}P(x,y,t)\right) \tag{12}$$

with the initial condition

$$P(x,y,0) = \delta(x-X) + \delta(y-Y) \tag{13}$$

The explicit solution for (12) with the initial condition (13) is obtained from (11) and it reads

$$P(x,y,t) = \frac{Ai\left(\frac{1}{3}\frac{3^{2/3}(-x+X)}{\eta_1^{1/3}t^{1/3}}\right) Ai\left(\frac{1}{3}\frac{3^{2/3}(-y+Y)}{\eta_2^{1/3}t^{1/3}}\right)}{3\eta_1^{1/3}\eta_2^{1/3}t^{2/3}} \tag{14}$$

In the case when $\eta = \eta_1 = \eta_2$, Equation 14 is reduced to

$$P(x,y,t) = \frac{Ai\left(\frac{3^{2/3}(-x+X)}{3\eta_1^{1/3}t^{1/3}}\right) Ai\left(\frac{3^{2/3}(-y+Y)}{3\eta_2^{1/3}t^{1/3}}\right)}{3\eta^{2/3}t^{2/3}} \tag{15}$$

and with the change of variable $\eta = \frac{\sigma^3}{t}$ Equation (15) takes the form

$$P(x,y,\sigma) = \frac{3^{1/3} Ai\left(\frac{3^{2/3}(-x+X)}{3\sigma}\right) Ai\left(\frac{3^{2/3}(-y+Y)}{3\sigma}\right)}{3\sigma^2} \tag{16}$$

The solutions (11) and (14) were derived using the Maple code in AppendixA[21]. Using Equation (5) it is possible to construct the Airy filter using the Maple code in AppendixB[21] with the Maple package *ImageTools*.

The Airy filter performs well as an edge detector. An example is showed at Figure 1.

Figure 2 compares the result of the application of the standard Sobel edge detector with the result of the application of the Airy edge detector. It is observed that the Sobel edge detector is more susceptible to the noise compared with the Airy filter. The application of the Sobel filter was performed using the program *ImageJ*.

Figure 3 shows the result of the application of the Canny edge detector to the images in Figure 2, it is observed that the image preprocessed with the Airy filter loses some information of the veins. Figure 3 was obtained using the program *ImageJ*.

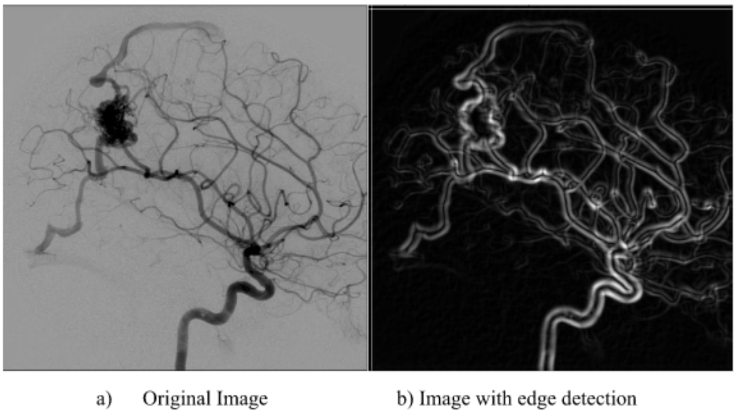


Figure 1: The result of an experiment of edge detection of an angiogram of AVM with a draining vein with the Airy filter.

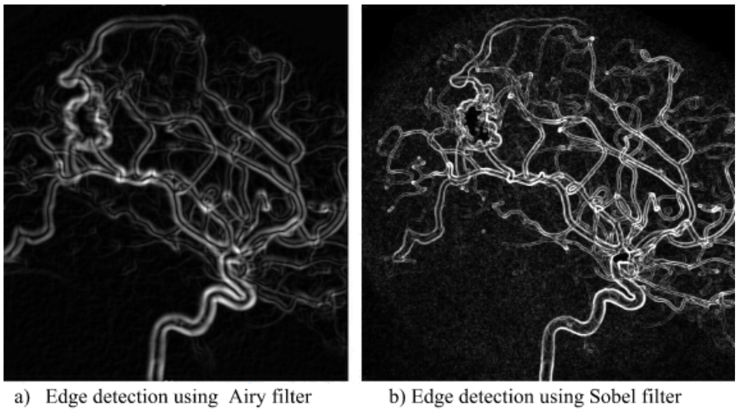


Figure 2: Comparing the results when the Airy filter and the Sobel filter are applied to the same image.

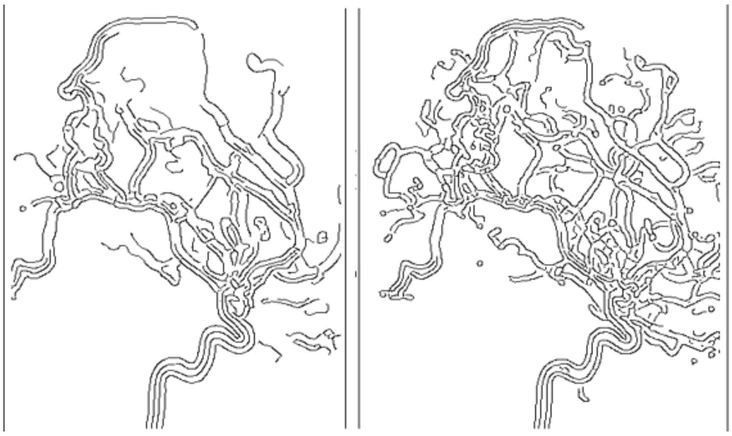


Figure 3: The result from the application of the Canny edge detector to the image in Figure 2.

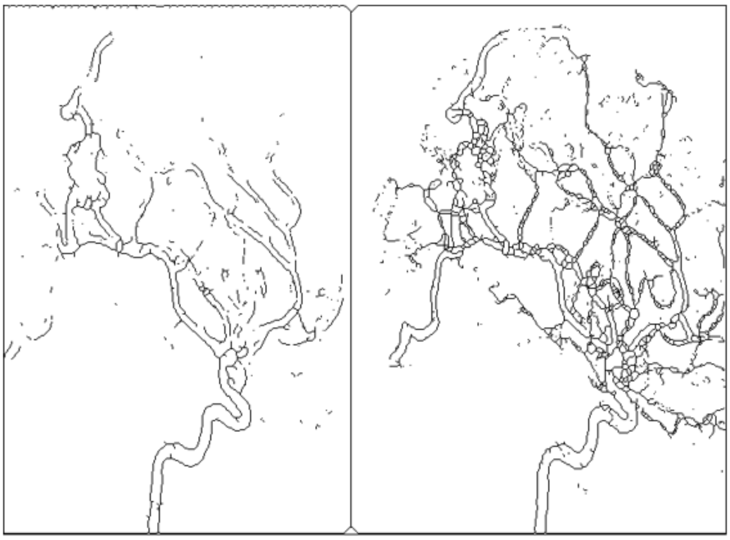


Figure 4: The result from the application of skeletonization to the image in Figure 2.

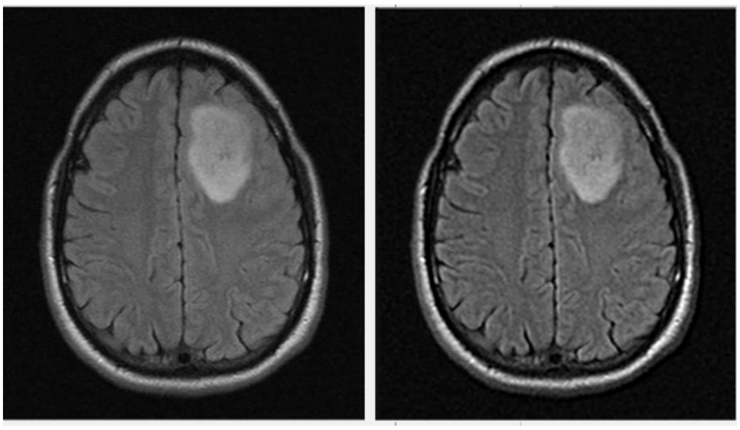
Figure 4 shows the result of the application of skeletonization to the images in Figure 2, where the skeletonization of the image with the Sobel filter provides more information than the one with the Airy filter. Figure 4 was obtained using the program *ImageJ*.

From fluid dynamics, the Buckmaster equation [17] is given by

$$\begin{aligned}\frac{\partial}{\partial t}u(x,y,t) = &\left(\frac{\partial^2}{\partial x^2}u(x,y,t)^4\right) + \left(\frac{\partial}{\partial x}u(x,y,t)^3\right)\\ &+ \left(\frac{\partial^2}{\partial y^2}u(x,y,t)^4\right) + \left(\frac{\partial}{\partial y}u(x,y,t)^3\right)\end{aligned} \tag{17}$$

Equation (17) is a non-linear equation and it is not possible to find an analytical solution. We construct a new filter named here the Buckmaster-Airy filter applying the spatial operator Buckmaster to solution (16). Then we have

$$\begin{aligned}BA = &\frac{8}{27\sigma^{10}}\left(3^{2/3}Ai\left(\frac{3^{2/3}(-x+X)}{3\sigma}\right)^4 Ai\left(\frac{3^{2/3}(-y+Y)}{3\sigma}\right)^4\right)\\ &+\frac{4}{81\sigma^{11}}\left(3^{1/3}(-x+X)Ai\left(\frac{3^{2/3}(-x+X)}{3\sigma}\right)^4 Ai\left(\frac{3^{2/3}(-y+Y)}{3\sigma}\right)^4\right)\\ &-\frac{18}{\sigma^7}\left(3^{2/3}Ai\left(\frac{3^{2/3}(-x+X)}{3\sigma}\right)^3 Ai\left(\frac{3^{2/3}(-y+Y)}{3\sigma}\right)^3\right)\\ &+\frac{4}{81\sigma^{11}}\left(3^{1/3}(-y+Y)Ai\left(\frac{3^{2/3}(-x+X)}{3\sigma}\right)^4 Ai\left(\frac{3^{2/3}(-y+Y)}{3\sigma}\right)^4\right)\end{aligned} \tag{18}$$



a) Original Image b) Processed Image

Figure 5: Results of the first experiment with the BA filter.

The filter in (18) is applied to a given biomedical digital image via convolution in Maple according with

$$Img(x, y, \sigma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} BA(x - \eta, y - \xi) Img(\eta, \xi) d\eta d\xi \tag{19}$$

For practical applications Equations (18) and (19) must be discretized and the Maple package *ImageTools* will be applied with the code in appendixC in the repository.

4 Results

Using the Buckmaster-Airy filter we obtain the preliminary results showed at Figures 5, 6, 7, 8, 9, 10, 11 and 12. These results are showing that the BA filter is a powerful tool of segmentation and extraction of vessels, providing a better extraction of the vessels of the biomedical image than other filters used in preprocessing. Many more experiments with the BA filter are required. Also it is important to compare the performance of the BA filter with other filters obtained from partial differential equations.

The new technique for vessel extraction proposed here is more powerful than the actually existing techniques such as pattern recognition techniques and artificial intelligence-based approaches. The reason is that our new technique is based on partial differential equations which are maybe the more powerful tool in digital image processing. The power of our new filter is amplified thanks the application of complex special functions of mathematical physics such as the Bessel function. With all these mathematical tools we can create a new filter which can be used for a better visualization of biomedical images and for many applications such as vessel extraction. Finally the application of computer algebra software provides a very powerful computational environment for discovering and prototyping new very important filters.

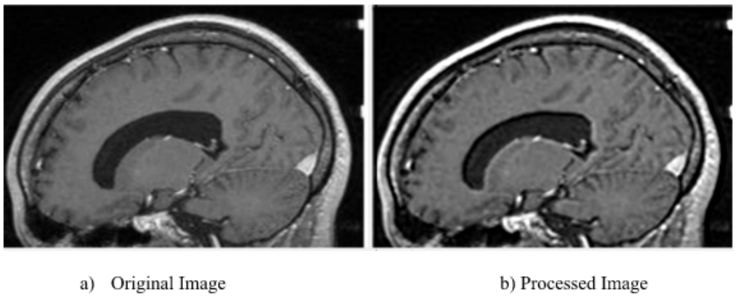


Figure 6: Results of the second experiment with the BA filter.

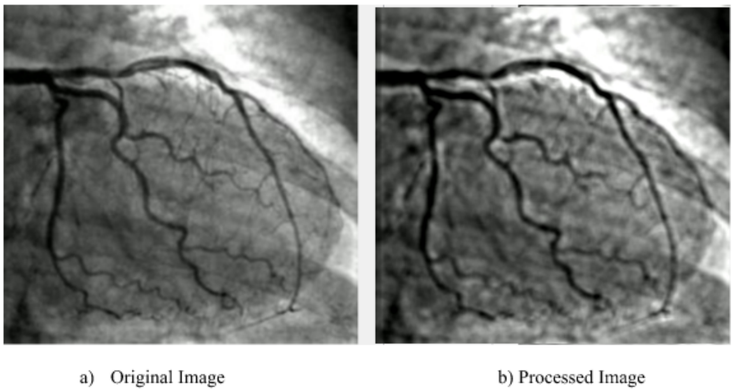


Figure 7: Results of the third experiment with the BA filter.

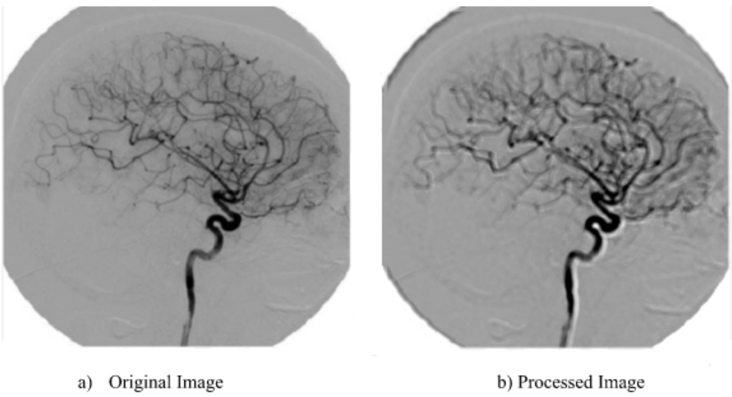


Figure 8: Results of the fourth experiment with the BA filter.

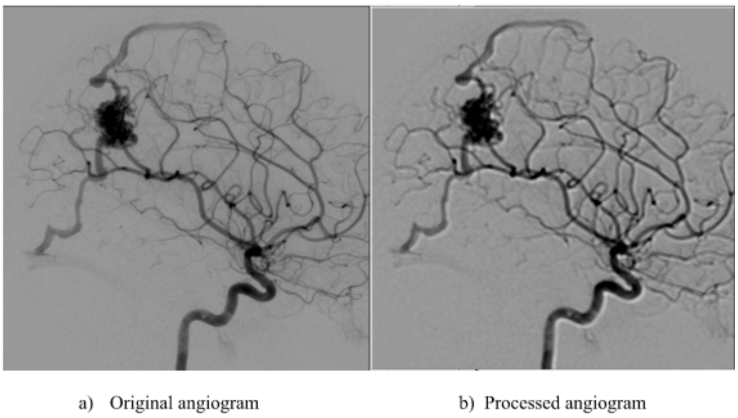


Figure 9: Angiogram of AVM with draining vein using the BA filter.

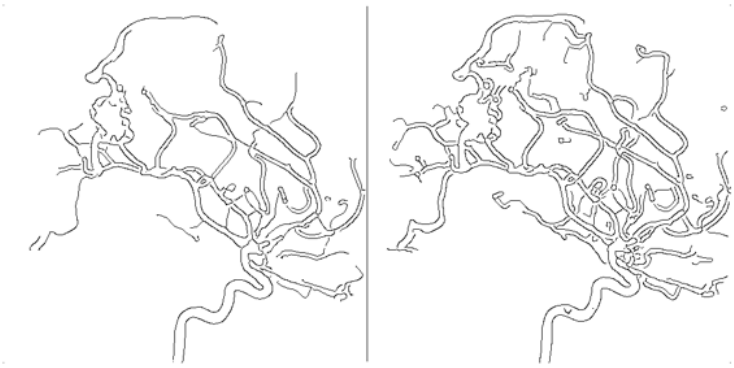


Figure 10: Canny edge detection of the image in Figure 9.

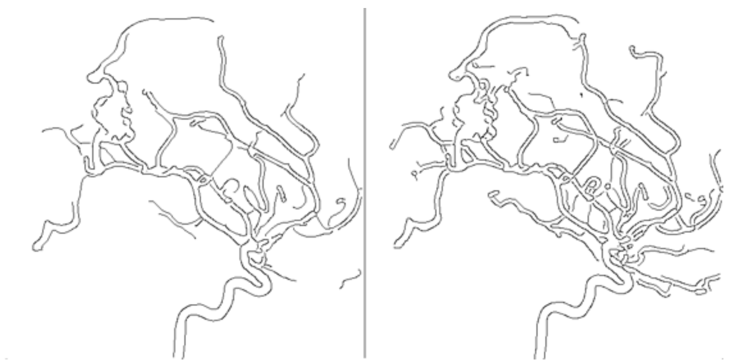


Figure 11: Canny edge detection after anisotropic diffusion of the image in Figure 9.

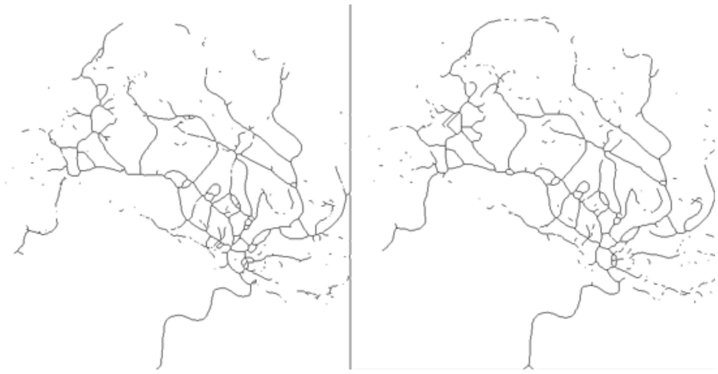


Figure 12: Skeletonization of the image in Figure 9.

Figures 10, 11 and 12 are results of the fifth experiment with the BA filter. Experiment of detection of brain aneurysms from an angiogram.

5 Conclusions

A new filter for vessel extraction were presented and tested. The new filter named Buckmaster-Airy filter is based on partial differential equations and it was designed and tested using the Maple package *ImageTools*. Some experiments were performed and the results were very good. The Buckmaster-Airy filter is able to produce an enhancement of the dendritic structures of the image without producing alterations on the edges of the filaments. As a line for future research is interesting to consider the possible combination of the Buckmaster-Airy filter with anisotropic diffusion and Canny filter with the aim to obtain a more powerful vessel extractor. Also is very interesting to consider the possibility to implement the Buckmaster-Airy filter as a new plugging in the program *ImageJ*; in such way that the combination Buckmaster-Airy filter + Anistropic diffusion + Canny filter will provide us a powerful vessel detector and extractor.

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